An Experimental Investigation of High Temperature Superconductors

Presented to the faculty of Lycoming College
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departmental honors in Astronomy/Physics

by
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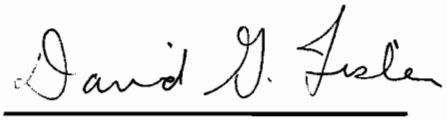
28 April 2004



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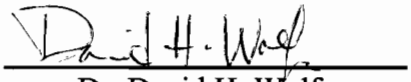

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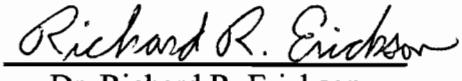

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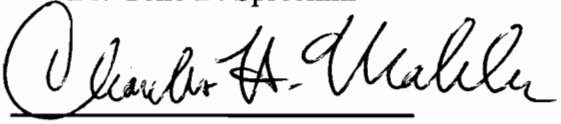

Dr. Charles H. Mahler

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April 28, 2004

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This project is dedicated to
Robert and Marguerite Benoit.
Without your love, guidance,
care, and support, none of this
would have been possible.

SECTION I: Introduction

A current trend in physics and technology in the world today is toward solid-state physics and superconductors. Whether in a newspaper, magazine, or professional journal, superconductors have been a topic of much public interest for the last twenty years. Despite being so popular for so long, it does not seem that the fascination people have with superconductors is beginning to fade. What is it about such a complicated topic that would bring so much attention to the often overlooked and otherwise unnoticed scientific community? The answer is actually very simple: The benefits to society. The impact that superconductors have on the average person's life today is unbelievable with applications ranging from transistors to MRI's. The future impact of commercially viable superconductors is even more staggering. Designs for magnetically levitated vehicles, electrical power distribution systems, and power generators that utilize superconductors are not just the fancy dreams of science fiction, but slowly becoming reality. The research and development of such beneficial systems for mankind provides ample justification for the necessary investigation and experimentation that will lead to a better understanding of what superconductors are how their applications can improve everyday lives.

What is a superconductor and how does it work? In order to answer that inquiry, two other questions must be addressed first: What does it mean to be an electrical conductor, and how does a material for that matter "conduct"? It is easiest to start by stating that all atoms in the universe are composed of elementary particles called protons, neutrons, and electrons, each of which has a property called charge associated with it. Charge is an intrinsic property of matter that affects the way that all matter interacts. Objects can be positively charged (like protons), negatively charged (electrons), or neutral (neutrons). In addition, objects with similar charges repel one another while objects with opposite charges are attracted to one another. Charge is measured in coulombs, where one coulomb is amount of charge that passes through a cross section of wire in one second when an electric current of one ampere flows in the wire.[1] The electrostatic force of attraction and repulsion between two objects is governed by Coulomb's Law, which can be expressed mathematically as:

[1] $$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

where q_1 and q_2 are the charges on the two objects, ϵ_o is a constant called the permittivity of free space, and r is the distance between the two objects. In an atom, the protons and neutrons are clumped together in the nucleus at the very center of the atom while the electrons occupy orbitals, or shells, surrounding the nucleus. Because the amount of positive charge on a proton is equal to the magnitude of negative charge on an electron, the electrons are attracted to the protons through Coulomb's Law. Thus, the attraction between electrons and protons is electrical in nature as opposed to gravitationally as in the case of the planets. The problem with that model is that classic electromagnetic theory predicts that an accelerating charge emits electromagnetic radiation. An electron orbiting a nucleus experiences a centripetal acceleration inward due to the electrostatic

[1] Halliday, D, Resnick, R & Walker, J. Fundamentals of Physics 6th Ed. John Wiley & Sons Inc. New York, NY. ©2001

force between the nucleus and the electron. Thus the electron should be constantly radiating energy and spiraling into the nucleus, a situation that does not occur. The only known physical mechanism that allows electrons to remain bound to a nucleus is the quantization of an electron's energy, i.e. the electrons surrounding the nucleus are only able to hold specific amounts of energy called quanta. The scientific theory that describes the behavior of quanta is called quantum theory and it plays a crucial role in the explaining the properties and behavior of superconductors*.

An electron bound to a nucleus can only hold quantized amounts of energy called energy levels. An electron can only gain energy if the energy it receives is enough to move to a higher energy level. Similarly, it can only lose energy if the energy given off is enough to drop the electron into a lower energy level. A simplified version of quantum theory predicts that two electrons are allowed to fill the first energy level, or shell since the atom is a three dimensional object. Eight electrons go in the second shell, eighteen in the third, and n^2 in the n^{th} level. The outermost shell that contains electrons in an atom is called the valence shell. The principle of least action says that every atom tries to achieve the lowest possible energy state, which will occur when an atom has filled its valence shell. Thus, atoms are trying to gain or lose electrons in an effort to achieve their lowest possible energy state. Since atoms with one, two or three electrons in their outer shell want to get rid of those electrons; they become very easy to remove from an atom. Conversely, atoms with five, six, or seven electrons in their valence shells try to gain electrons.

An electrical current is a moving electric charge and a moving electron naturally fits that description as it holds a negative charge of magnitude $e = 1.602 \times 10^{-19}$ Coulombs. In certain solid materials called conductors, electrons (i.e. charge) can move about from atom to atom with relative ease. That is why metals are good conductors; when the atoms of a specific metal bond together to form a solid, all the atoms have valence electrons that they want to get rid of, thus the electrons can leave the atom with relative ease. Insulators are materials that do not give up electrons readily because they have their valence shells almost filled or are already in their lowest possible energy state. Those materials try to retain electrons, making it very hard for a current to flow. Semiconductors, such as silicon are materials that fall somewhere in between conductors and insulators; they allow a current to pass through them albeit a very small one.

Since ancient times people noticed that electrical phenomena were similar to magnetic phenomena, but it was not until 1819 when Hans Christian Oersted was performing experiments using electrical currents. During one trial, he accidentally placed his compass too close to a wire that had electricity flowing through it. Investigating the matter further, Oersted found that when he moved the compass close to the wire, the needle always pointed perpendicular to the wire. The only explanation for the behavior of the compass was that the current in the wire generated a magnetic field that aligned with the magnetic field of the compass. Michael Faraday reasoned that if a current could generate a magnetic field then a magnetic field should be able to generate an electrical current in a wire. He termed the process by which that happens induction, i.e. a magnet induces a current to flow. Induction is the basic principle behind an electromagnet;

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* The physics of superconductivity is based on how electrons behave inside materials and interact with many atoms. For a description of electron interactions with one atom present, see the Franck-Hertz Experiment in Appendix A.

current in the coils of a wire wrapped around a conducting rod induces a magnetic field inside the rod and turns it into a magnet. Faraday realized the impact of how his theory could be applied to real world applications and built the world's first crude electrical motor. Soon after Faraday's discovery, the industrial age began and electrical power was distributed to homes and factories. With the increased demand for electricity came an increased need to generate and distribute it.

There was one problem with large power distribution systems; they lost large amounts of energy to the surrounding environment in the form of heat. The cause of the energy loss was electrical resistance. Resistance is exactly what it sounds like, an opposition to electrical current. Most conductors experience resistance according to a relationship called Ohm's Law: The potential difference (voltage) applied across a conductor is directly proportional to the current that can pass through that conductor. The electrical potential is the energy per unit charge at a given point. A difference in electric potential between two points causes electrons to move because all objects have a tendency to be in the lowest possible energy state. Thus, if there is a difference in potential energy, electrons will flow from a region of higher potential to lower potential, generating a current. Of course, as the electrons flow, they interact with other electrons as well as nuclei of the atoms that make up a conductor through collisions. Energy is lost in the collisions and is released as heat. More electron collisions result in a greater resistance to electrical current and the object through which electricity is flowing will have a greater resistance. That creates a problem for conducting wires because the greater the current that is passed through a wire, the more energy is released as heat and the hotter the wire becomes. If the wire was to get too hot; then it would melt and the flow of electricity would stop. To further complicate things, objects that are good electrical conductors are also good thermal conductors. As resistance releases more heat, the heat flows through the conductor, giving more energy to the atoms and electrons inside, further driving up the resistance of the object, which making it even harder for a current to flow.

For this reason, in 1911 Heike Kamerlingh Onnes began experiments to test the electrical properties of various metals at low temperatures using liquid helium (~ 4 K). It was his hypothesis that in a lower state of energy, there would be less resistance to the flow of electricity in metals. Onnes had expected to find resistance drop exponentially with temperature and then eventually level off to a limiting resistance that would never be able to be overcome. He observed that indeed resistance dropped as a function of temperature but he also observed something unexpected and astonishing. To Onnes' surprise, he found that around 4 K, mercury completely lost all resistance as did various other metals. He even formed a ring out of mercury, submerged it in liquid helium, and passed a current through it. Onnes then allowed it to sit like that, while making sure there was always plenty of coolant around the mercury. After a year, he measured the current in the ring and found that it had persisted virtually undiminished![2] Often scientists will talk about physical phenomena approaching zero, or being negligible, but very rarely do they find something as remarkable as Onnes had. Onnes termed his new discovery superconductivity and described the temperature at which a superconductor lost resistivity as the critical temperature (T_c). The scientific community realized the

[2] Mayo, Jonathan L. Superconductivity: The Threshold of a New Technology. Tab Books Inc. Blue Ridge Summit, PA. ©1988.

importance of Onnes' discovery and quickly rushed to find materials with higher critical temperatures. Liquid helium is relatively expensive to produce so the prospect of finding a commercially viable superconductor that had a higher T_c was very lucrative.

All the known metals were subjected to test after test and when they were exhausted, combinations of different metals were tested to see if any would yield significant results. Unfortunately, progress towards raising the critical temperature stalled for 60 years before IBM researchers Alex Müller and Georg Bednorz had constructed a ceramic superconductor with a critical temperature of 30 K[3]. Soon after, in 1987, Paul Chu and a research team at the University of Houston reported developing a material that would become superconducting at 98 K, which could be cooled using liquid nitrogen, a safer and more readily available cryogen. With the development of these new "high temperature" superconductors, the field of solid-state physics was rejuvenated with the hope of inventing materials that would become superconducting at room temperatures. Even at the dawn of the 21st century, resistance still limits not only power lines, but the size of processor chips for computers and other electrical devices as well. This investigation was carried out with two needs in mind: The obvious need for the development of materials to improve the quality and efficiency of electrical devices, and the need to expose future generations of scientists to such an important field of study.

[3] Mayo

SECTION II: Theory

Crystallography

The atoms in a solid are held in place by the coulomb interaction in such a way that they form a self-repeating geometrical shape called a lattice. A series of lattices can be put together to form a crystal. The study of lattices and crystals and how they form is called crystallography, a science that is very relevant to superconductors. The structure and composition of solid crystals has a very large effect on how electrons can be transported through materials, in other words, the molecular make up of a crystal affects the conductivity that the crystal will have. The structure of a perfect crystal is defined in terms of the lattice with an atom or a group of atoms attached to every lattice point. The group of atoms is called a basis, which when repeated in space, forms the crystal structure.[4] Lattice points can be connected to one another through the primitive translation vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ so that the lattice looks exactly the same for a point $\mathbf{r}$ as it does for a point $\mathbf{r'} = \mathbf{r} + A\mathbf{a}_1 + B\mathbf{a}_2 + C\mathbf{a}_3$, where A, B, and C are arbitrary integers. The

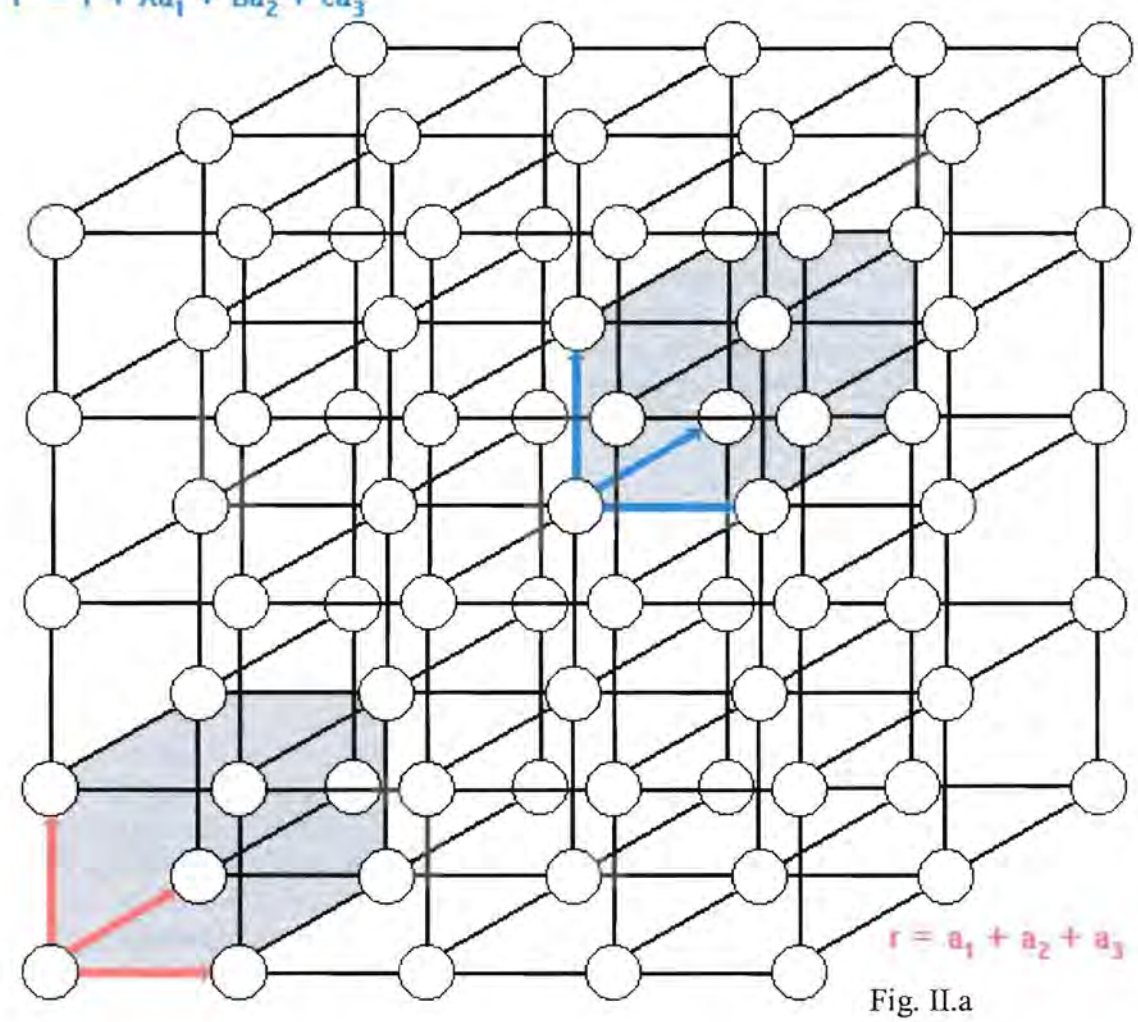


Fig. II.a

[4] Kittel, Charles. Introduction to Solid State Physics. John Wiley & Sons. New York, NY. ©1996

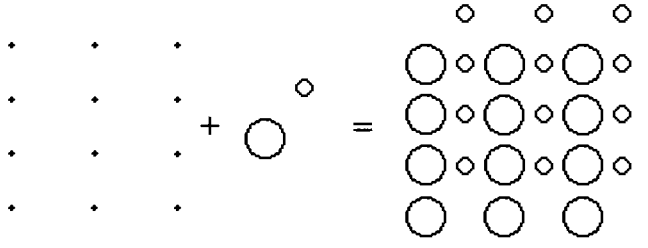
magnitudes of $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ correspond to the inter-atomic spacing in the direction of $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ respectively. The primitive translation vectors define the unit cell, which serves as the basic building block for the crystal. It is important to note that the choice of the primitive translation vectors is arbitrary, but often vectors are chosen that will be able to exploit the symmetry of a given structure. The vector $\mathbf{T} = A\mathbf{a}_1 + B\mathbf{a}_2 + C\mathbf{a}_3$ is called the (crystal) translation vector and defines the translation of the unit cell. Reflection and rotation are some other symmetry operations that can be applied to the unit cell.

Figure II.a demonstrates how **r** defines the unit cell and the translation by **T** for a cubic lattice. The figure also shows how the lattice will look identical when viewed from either **r** or **r'**. That will be the case when the crystal is so large that atomic scales are insignificant and the lattice looks as if it extends to infinity in all directions. The cell that contains the smallest possible volume of any cell that can be constructed in the lattice is the unit cell. That is an important aspect of the unit cell because it is necessary for the unit cell to be the building block of the lattice. The minimum volume can be defined through vector calculus to be the volume of a parallelepiped defined by the primitive translation vectors:

[II.1] $$V = \left|\vec{a}_1 \bullet \vec{a}_2 \times \vec{a}_3\right|$$

There is only one lattice point per primitive unit cell, meaning that if a parallelepiped with eight corners has a lattice point at each corner, then each lattice point would be shared among eight adjoining cells so that the total number of lattice points in one cell would be
$8(1/8) = 1$. Similarly, the basis associated with a primitive cell is called a primitive basis[5], because no other basis will contain fewer atoms. The basis may consist of one atom, or a molecule that is placed at every point within the lattice structure. Figure II.b shows how a simple two-dimensional lattice can be combined with a two-atom molecule to form a simple crystal structure.

Fig II.b



It turns out that not every lattice can generate a crystal structure because in order to do so, the lattice must satisfy certain collections of symmetry operations, such as reflection across a plane and rotation about an axis. In three dimensions, fourteen basic lattices fulfill the symmetry requirements and they can be reclassified into seven basic cells because of the similarity in symmetry between many of the structures. The systems are: triclinic, monoclinic, orthorhombic, tetragonal, cubic, trigonal, and hexagonal. As an example, the cubic group includes three structures known as the square cubic, the face centered cubic (fcc) and the body centered cubic (bcc). The face centered cubic is a cube

[5] Kittel

that has extra lattice points centered on each of its six faces while the body centered cubic has a lattice point in the center of the cube.

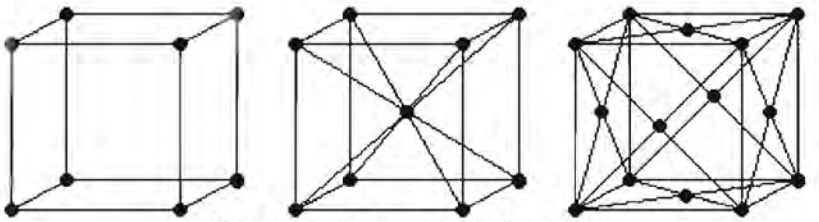


Fig. II.c: simple cubic, bcc, fcc

The position of a point inside a cell is defined as multiples of the primitive translation vectors with notation that differs from the standard expression for a vector. The origin is usually picked to be an arbitrary corner of the cell and positions are measured from there. In the case of a cube, the center would have the coordinates (½, ½, ½) and the centers of some of the faces would be (0, ½, ½), (½, 0, ½), and (½, ½, 0). The orientation of planes within a crystal is handled in a similar manner, though it turns out to be more convenient to describe the planes by using the following technique:

1) Find the intercepts of the plane in terms of the lattice constants $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$.
2) Take the reciprocals of those numbers and reduce them to the smallest three integers having the same ratio.
3) The result is written in parenthesis (hkl) and is called the index of the plane.

Take for example, a plane that has the intercepts (3, 2, 4); the reciprocals are ⅓, ½, ¼ and the smallest three integers having the same ratio are 4, 6, 3; thus (hkl) = (463). If one of a plane's intercepts happens to cross an axis on the negative side of the origin, then the negative sign is denoted by a bar over the appropriate index. For example, a plane parallel to the $\mathbf{a}_2\mathbf{a}_3$ axes that intersects the $\mathbf{a}_1$ axis at a_1 will have the coordinates $(\bar{1}\,00)$.

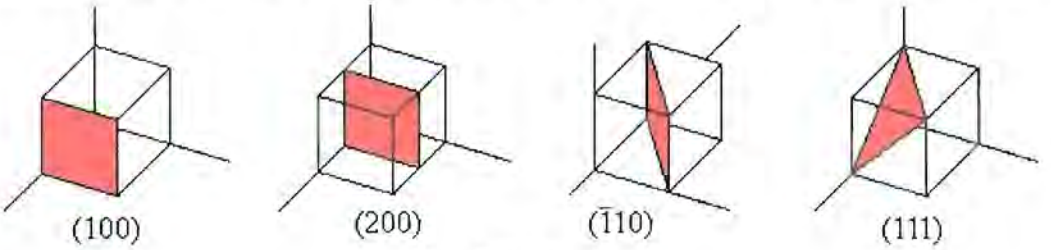


Fig II.d

Reciprocal Space and Diffraction

Every crystal can be represented using one of two lattices, the crystal lattice defined in the previous section and a reciprocal lattice, which is a representation of the real lattice in reciprocal, or "k" space. A microscopic image, if properly resolved, would be a map of the crystal structure in real space, while a diffraction pattern is a map of the reciprocal lattice in k-space.[6] The term k-space is derived from the fact that observations are measured in units of 1/distance, which are the units on the wave number $k = 2\pi/\lambda$. Momentum is related to the wave number through $\mathbf{p} = \hbar\mathbf{k}$, so k-space is very similar to momentum-space and differs only by a scale factor. The two lattice representations are related by defining the primitive vectors of the reciprocal lattice $\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{b}_3$ to be:

[II.2] $$\vec{b}_1 = 2\pi\frac{\vec{a}_2\times\vec{a}_3}{\vec{a}_1\bullet\vec{a}_2\times\vec{a}_3};\vec{b}_2 = 2\pi\frac{\vec{a}_3\times\vec{a}_1}{\vec{a}_1\bullet\vec{a}_2\times\vec{a}_3};\vec{b}_3 = 2\pi\frac{\vec{a}_1\times\vec{a}_2}{\vec{a}_1\bullet\vec{a}_2\times\vec{a}_3}$$

The main purpose in defining the reciprocal lattice vectors is to simplify calculations involving wave vectors, which are represented in k-space. The new primitive vectors form the k-space counterpart to the translation vector **T** in real space called the reciprocal lattice vector $\mathbf{G} = u_1\mathbf{b}_1 + u_2\mathbf{b}_2 + u_3\mathbf{b}_3$, where u_1, u_2, u_3 are arbitrary constants.

The set of reciprocal lattice vectors **G** for any particular crystal determines the allowed x-ray diffractions through that crystal. That result can be shown, by recognizing that any local property of a crystal will be invariant under any translation by **T**. This could include a property such as charge concentration, magnetic moment density, or electron number density. Because the latter is a function of **r**, the distance from the origin, the electron number density can be expressed as $n(\mathbf{r})$, but because it is invariant under any translation **T**, it must be periodic with **T** and satisfy the condition:

[II.3] $$n(\mathbf{r}) = n(\mathbf{r} + \mathbf{T})$$

a condition that is an ideal situation for Fourier analysis. A Fourier transform will turn a function of real space into a function of reciprocal space, resulting in equation II.4, which must be invariant under any crystal translation **T**.

[II.4] $$n(\vec{r}) = \sum_{\vec{G}} n_{\vec{G}} e^{i\vec{G}\bullet\vec{r}}$$

where n_G is the complex Fourier coefficient of the transformation. Given II.4, making the correct substitutions yields:

[II.5] $$n(\vec{r}+\vec{T}) = \sum_{\vec{G}} n_{\vec{G}} e^{i\vec{G}\bullet(\vec{r}+\vec{T})} = \sum_{\vec{G}} n_{\vec{G}} e^{i\vec{G}\bullet\vec{r}} e^{i\vec{G}\bullet\vec{T}} = \sum_{\vec{G}} n_{\vec{G}} e^{i\vec{G}\bullet\vec{r}} = n(\vec{r})$$

because $\exp(2\pi i * q) = 1$, where q is any integer. Since $p = u_1A + u_2B + u_3C$ is a combination of integers, p is an integer so:

[II.6] $$e^{i\vec{G}\bullet\vec{T}} = e^{i2\pi(u_1A+u_2B+u_3C)} = 1$$

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[6] Kittel

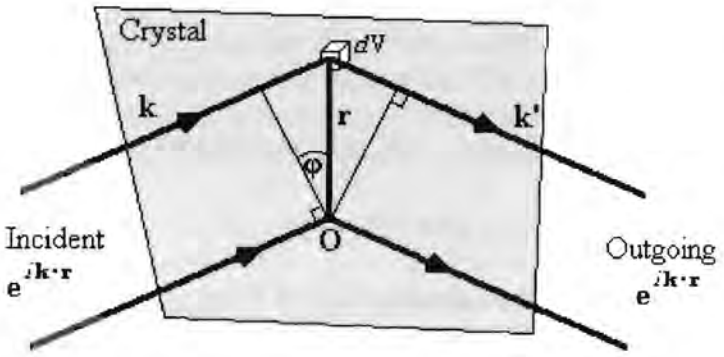


Figure II.e

and the result is an equation that satisfies II.3. Figure II.e shows the difference in phase between two wave vectors scattered from two different volume elements a vector **r** apart. The difference in phase angle between the incident waves at O and **r** is $(2\pi r\sin\phi)/\lambda = \bar{k} \bullet \bar{r}$. The difference between the phase angle in the diffracted waves leaving O and **r** is $-\bar{k} \bullet \bar{r}$. Thus the total difference in the phase angle between the incident and reflected waves will be $(\bar{k} - \bar{k}') \bullet \bar{r}$ and the wave scattered from dV at **r** has a phase factor relative to the wave scattered from a volume element at O:

[II.7] $$e^{i(\bar{k}-\bar{k}')\bullet\bar{r}} = e^{-i\Delta\bar{k}\bullet\bar{r}}$$

Figure II.f demonstrates how **k** can be mathematically transformed into **k'** through the addition of a vector **Δk**, which is the change in the wave number of the scattered x-rays.

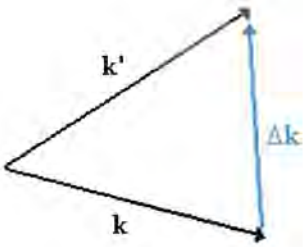


Fig. II.f

[II.8] $$\mathbf{k} + \Delta\mathbf{k} = \mathbf{k'}$$

Thus, the term **k** – **k'** in the argument of II.7 can be replaced with –**Δk**. The amplitude A of the x-ray scattered from a volume element is proportional to the local electron concentration at the atom. Therefore, the total amplitude of the scattered wave is proportional to the integral over the entire crystal of n(**r**)dV multiplied by the phase factor II.7.

[II.9] $$A = \int dV n(\bar{r}) e^{i(\bar{k}-\bar{k}')\bullet\bar{r}}$$

Substituting II.4 for n(**r**) of equation II.9 results in an equation for the scattering amplitude:

[II.10] $$A = \sum_{\bar{G}} \int dV n_{\bar{G}} e^{i(\bar{G}-\Delta\bar{k})\bullet\bar{r}}$$

this will be maximized when the argument of the exponential is zero, or when:

[II.11] $$\Delta\mathbf{k} = \mathbf{G}$$

meaning that the diffraction maxima will occur when the change in wave vector is equal to the reciprocal lattice vector. This condition for diffraction can be rewritten by

substituting **G** for $\Delta\mathbf{k}$ in [II.8] and recognizing that the scattering of a photon off an atom is an elastic process for which the energy hυ is conserved. Thus, the frequency of the incoming wave must be the same as that of the outgoing wave, requiring that the magnitudes of the wave vectors **k** and **k'** must be equal, and so are the squares of their magnitudes. Therefore, using the substitutions above:

$$|\mathbf{k}'|^2 = \mathbf{k}'^2 = (\mathbf{k} + \mathbf{G})^2 = \mathbf{k}^2 \rightarrow \mathbf{k}^2 - (\mathbf{k} + \mathbf{G})^2 = 0$$

Using the fact that if **G** is a reciprocal lattice vector, then **–G** will be one as well and the diffraction condition can be restated as:

[II.12] $$2\vec{k} \bullet \vec{G} = \vec{G}^2$$

Diffraction plays a large role in solid-state physics because of its role in determining the crystal structure and inner workings of a particular material. The dependency on interpreting results lead to the development of the reciprocal lattice

Brillouin Zones

Léon Brillouin used the diffraction condition to define a unit cell in the reciprocal lattice, by rearranging [II.12]:

[II.13] $$\vec{k} \bullet \frac{1}{2}\vec{G} = \left|\frac{1}{2}G\right|^2$$

Working in reciprocal space, a vector **G** can be constructed from the origin to a reciprocal lattice point close to the origin and a perpendicular bisector is drawn through **G**. The plane then forms a boundary for the cell and any **k** that is drawn from the origin to the plane will satisfy [II.13] as is demonstrated in Figure II.g. The set of all of the planes that can be constructed by bisecting a vector from the origin to the nearest reciprocal lattice points forms the First Brillouin Zone. The set of planes that can be constructed using the second closest neighbors to the origin is called the second Brillouin zone, and so fourth. If one considers reciprocal space in one dimension, then [II.2] shows that the magnitude of a basis vector **b** will be: $b = 2\pi/a$, where a is the inter-atomic spacing. The shortest reciprocal lattice vectors from the origin are **b** and **–b**. Constructing perpendicular bisectors through **G** = **b** results in the Brillouin zone boundaries for which, the shortest distance between the origin and the zone boundaries will be $(\frac{1}{2})(2\pi/a) = \pi/a$. Therefore, in one dimension, the zone boundaries will be at $k = \pm\pi/a$. Although seemingly unimportant at this stage, the Brillouin zone is a construction that allows physicists to simplify equations by working in reciprocal space, making the Brillouin zone the elemental building block for solid-state physics.

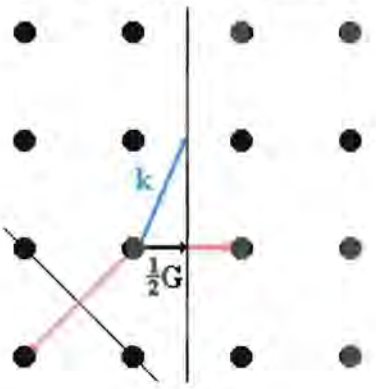


Fig. II.g

Free Electron Model

Now that a foundation for crystal structures has been introduced, the behavior of electrons within the crystal can now be discussed. To ease the transition into this topic, it is often best to approximate the behavior of electrons using what is known as the Free Electron Model. The model treats electrons in a lattice as a gas, which has no potential acting on it. Although an oversimplification of what is physically going on, the model accurately predicts the kinetic behavior of conduction electrons, or electrons that can move freely throughout the lattice. It also describes how valence electrons can leave their individual atoms and become conduction electrons. There is only one place to begin a discussion on the electron, because of its small size, the Schrödinger equation for a free particle in three dimensions:

[II.14] $$\mathscr{H}\Psi = E\Psi \rightarrow -\frac{\hbar^2}{2m}\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\Psi(\vec{r}) = E_k\Psi(\vec{r})$$

Assuming the electrons are confined to a cube with the length of its sides equal to `l`, the wave function then must be periodic in `l` and solutions take on the form of a plane wave:

[II.15] $$\Psi(\vec{r}) = Ae^{i\vec{k}\bullet\vec{r}}$$

where A is a constant and $k_x = 2n_x\pi/l$, $k_y = 2n_y\pi/l$, and $k_z = 2n_z\pi/l$. The numbers n_x, n_y, and n_z are integers known as the quantum numbers and represent the energy state, or orbital of the electron. [II.15] can be used substituted back into the Schrödinger Equation to find an equation for the kinetic energy of an electron in a given orbital.

[II.16] $$E_k = \frac{\hbar^2}{2m}\left(k_x^{\,2}+k_y^{\,2}+k_z^{\,2}\right) = \frac{\hbar^2}{2m}k^2$$

For the ground state, or lowest energy state of an atom, the orbitals surrounding the nucleus may be represented as points inside a sphere in reciprocal space. The Pauli exclusion principle states that no two electrons in an atom can have the same four quantum numbers (n, l, m_l, m_s) at the same time. The first three numbers correspond to the orbital that the electron fills while the fourth quantum number corresponds to an intrinsic property of electrons called spin. Because of the exclusion principle, only two electrons with opposite spins can occupy the same orbitals in an atom. Therefore, electrons can only orbit a nucleus according to very strict rules. In general, if one had a nucleus of atomic number Z, and a pile of Z many electrons, and wanted to place the electrons around the nucleus, then one would have to do so in the following way. Two electrons are placed in the first orbital, one with spin up and one with spin down. Then two go in the second orbital, and two in each successive orbital on outward until all the electrons have been placed. Because there are no vacant orbitals in between any filled orbitals the sphere is completely filled and the atom in its ground state. The surface of the sphere is called the Fermi surface and it separates the filled orbitals from the unfilled ones. The Fermi surface also has a specific energy associated with it called the Fermi

energy (E_F), which is the energy associated with the outermost electron in the ground state of the atom. Such an electron will have a wave vector with magnitude k_F so that:

[II.17] $$E_F = \frac{\hbar^2}{2m} k_F^{\,2}$$

Remember that the sphere that defines the Fermi surface is in reciprocal space, so the radius of the sphere will be k_F, and its volume will be $(4/3)\pi k_F^{\,3}$. There is only one distinct set of principle quantum numbers n_x, n_y, n_z for a volume $(2\pi/1)^3$ in reciprocal space and two electrons can share the same set of principle quantum numbers provided they have different spin quantum numbers. Thus the total number of electrons inside the Fermi sphere will be the number of electrons per unit volume multiplied by the volume of the sphere:

[II.18] $$N = 2\frac{(4/3)\pi k_F^{\,3}}{(2\pi/\ell)^3} = \frac{V}{3\pi^2} k_F^{\,3} \rightarrow k_F = \left(\frac{3\pi^2 N}{V}\right)^{1/3}$$

where $V = l^{\,3}$, the volume of the atom in real space. This result may be substituted into [II.17] for an expression that shows the dependence of the Fermi energy on the electron concentration N/V:

[II.20] $$E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$$

Electrical Conduction

An electrical conductor is a material in which charge, either electrons or positive holes, can move freely. The free electron model can provide a reasonable approximation of the motion of electrons in an external electric field. Quantum mechanics requires that the momentum of a free electron is related to its wave vector through the equation $\mathbf{p} = \hbar\mathbf{k}$. In an electric field **E**, the force on an electron is $e\mathbf{E}$, and Newton's second law states:

[II.21] $$\vec{F} = \hbar\frac{\partial \vec{k}}{\partial t} = -e\vec{E}$$

If there are no collisions between particles inside the Fermi sphere, if a constant electric field is applied from a time 0 to a time t, then each electron in the Fermi sphere will be displaced at a uniform rate, and [II.21] can be integrated to:

[II.22] $$\mathbf{k}(t) - \mathbf{k}(0) = \delta\mathbf{k} = -e\mathbf{E}t/\hbar$$

Thus the entire sphere has been shifted an amount $\delta\mathbf{k}$. In actuality however, the electrons of the Fermi Sphere will collide with atoms, impurities, lattice imperfections, lattice vibrations, and other electrons. The collisions cause a decrease Fermi sphere shift and hence reduce the acceleration of the electrons. For a period of time τ between collisions,

the Fermi Sphere will be unaffected and its movement can be described using [II.22]. By converting wave vector to momentum through $\mathbf{p} = \hbar\mathbf{k} = m\mathbf{v}$ and assuming the initial velocity of the sphere was zero, a formula for the velocity of the electrons in the material can be derived.

[II.23] $$\mathbf{v_d} = e\mathbf{E}\tau/m$$

This velocity is called the drift velocity and it represents the average flow of electrons through a conductor. If the electric field is turned off, then at any particular time, a snapshot of the motion of the electrons would reveal each electron moving in a random path due to all of the possible interactions within the medium as in Figure II.f1. All of the random motions combine so that the average velocity of all the electrons is effectively zero. Turning on the electric field will give an overall orientation to the velocity of the electrons, which statistically averages to be the drift velocity: Figure II.f2.

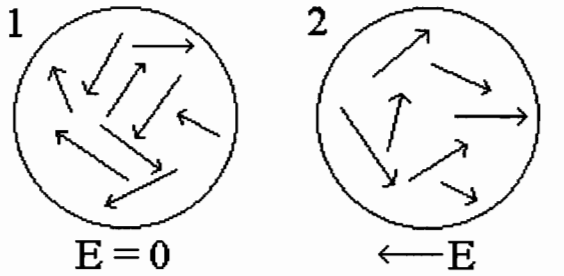


Fig II.f

The current density is defined to be the current per unit area within the conductor. If a constant electric field is set up inside the conductor and there are n electrons per unit volume, then the current density can be stated mathematically:

[II.24] $$\mathbf{J} = ne\mathbf{v_d} = ne^2\mathbf{E}\tau/m = \sigma\mathbf{E}$$

where $\sigma = ne^2\tau/m$ is defined to be the conductivity of the material. For a given electric field, by [II.24] a higher conductivity means a greater current density in the conductor, so obviously materials with a high conductivity are very good conductors. Similarly, a quantity called resistivity, denoted by $\rho = 1/\sigma$, is a measure of how poor a conductor a material will be. The resistance of an object is directly related to the resistivity of the material that makes up that object, i.e. resistance is a bulk property and resistivity is a microscopic property, but the two refer to the same phenomena. In a superconducting pellet, in the superconducting state, the resistance of the superconductor is zero; hence, the resistivity of the material that makes up the pellet is also zero. The process by which that happens will be discussed in due time, but first, a few more topics need to be covered to lay the proper foundation for such a discourse.

Energy Bands

The free electron model accurately predicts many observed phenomena, but it is severely limited by one simple fact; the electrons in a solid are not free, they experience a periodic potential generated by the atoms in the lattice. It will be shown that the potential gives rise to forbidden zones called band gaps, which are values of energy that electrons cannot have. The energies that electrons can have are called energy bands and the amount of electrons that fill the bands actually define whether a material will be a conductor, insulator or semiconductor. The band structure can be explained by the Nearly Free Electron Model, which is heavily dependent on Bragg reflection. Recall equation [II.13], the condition for a diffracted wave of wave vector **k**. In one dimension the equation will reduce to:

[II.25] $$k = \pm \tfrac{1}{2}G = \pm n\pi/a$$

where a is the distance between atoms in the lattice and $G = 2\pi m/a$. The region in k-space between $-\pi/a$ and π/a turns out to be the first Brillouin zone. Electrons traveling through the crystal take on wave vectors in the form of traveling waves according to [II.15]. In one dimension a traveling wave at the Brillouin zone boundaries ($k = \pm \pi/a$) will be Bragg reflected back on itself, so inside the zone boundaries there are actually two traveling waves $e^{i\pi x/a}$ and $e^{-i\pi x/a}$. Those two waves can combine in two waves to set up a standing wave within the zone that can be expressed as either:

[II.26]
$$\Psi(+) = e^{i\pi x/a} + e^{-i\pi x/a} = 2\cos(\pi x/a)$$
$$\Psi(-) = e^{i\pi x/a} - e^{-i\pi x/a} = 2i\sin(\pi x/a)$$

The two standing waves concentrate electrons in different regions, as can be seen by taking the probability density $\Psi^*\Psi$ or both functions:

[II.27a] $$|\Psi(+)|^2 = 4\cos^2(\pi x/a)$$

[II.27b] $$|\Psi(-)|^2 = 4\sin^2(\pi x/a)$$

Equation [II.27a] is maximized when x = 0, a, 2a,... which is the location of the atomic cores generating the potentials. The term core is used because the nucleus of the atom could still contain electrons and hold a net positive charge that would generate potential energy for an electron. Equation [II.27b] then concentrates electrons away from the cores. The electrons that are closer to the positive cores will have less potential energy than the electrons concentrated away from the cores, creating an energy gap between the wave functions. Electrons below the gap have the wave function $\Psi(+)$ and electrons below the gap have the wave function $\Psi(-)$.

However, the periodic potential caused by the lattice has not yet been taken into account. The next step is then to develop what is known as the Kronig-Penny model for a potential due to fixed ion sites separated by a distance a. From basic physics it is known

that the potential energy of one point charge due to a another is proportional to 1/r, where r is the distance between the charges. In a periodic potential, the individual potentials of each core will overlap to form one resultant potential that resembles the one-dimensional periodic potential configuration of Figure II.g. Of course, a potential such as this

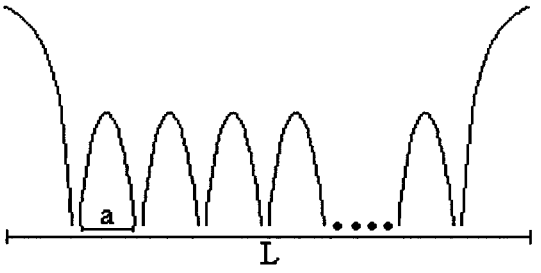


Fig. II.g

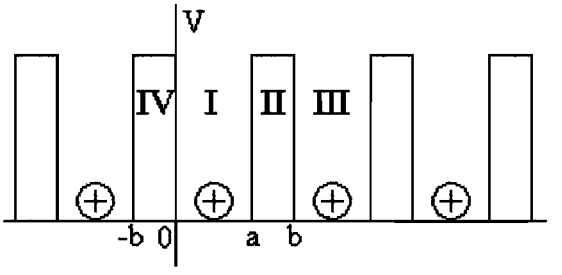


Fig. II.h

complicates calculations so two simplifying assumptions are made that bring the potential to look something like the repeating square wells of Figure II.h. The first is that the length of the whole crystal (L) is much greater then the size of the inter-atomic spacing, so big, that the ends of the crystal are essentially at infinity. This removes the loss of periodicity at the ends of the lattice. Fortunately, that is a reasonable simplification since the distance between atoms is on the order of 10^{-10}m, and the effect of the potential at the ends in reality has little effects on the transport properties of an electron inside the crystal. Turning the potential wells into square potentials is the second simplification. One would think that such a process would seriously affect the accuracy of the theory, but surprisingly, experimental results very closely match the Kronig-Penny model, justifying the deviation from the actual potential.

Naturally the Kronig-Penny model starts with the Schrödinger equation, which again for simplification reasons, will only be considered in one dimension. The difference between that and the Free Electron Model is that the energy now also takes into account a periodic potential, V(x) = V(x + a), so the Hamiltonian becomes:

[II.28] $$H = \frac{\hbar^2 k^2}{2m} + V(x)$$

F. Bloch showed that the eigenfunctions of this Hamiltonian with the restriction of a periodic potential are of the form:

[II.29] $$\Psi(x) = u(x)e^{ikx}$$

where u(x) = u(x + a). Functions with this behavior are called Bloch functions and they satisfy the periodicity restraint by having u(x) be cyclic. Thus, the solution in region I must be related to the solution in the region III and the solution in region II must be related to the solution in region IV and so on. This is known as Bloch's Theorem and can be stated mathematically for regions II and IV:

[II.30] $$\Psi(a \le x \le a+b) = \Psi(-b \le x \le 0)e^{ik(a+b)}$$

Because an electron could be traveling either to the right or to the left, the solution to the Schrödinger equation for each region is a superposition of traveling waves:

[II.29]
$$\text{Region I, } E = \frac{\hbar^2 k_1^{\,2}}{2m}: \qquad \Psi_I = Ae^{ik_1 x} + Be^{-ik_1 x}$$
$$\text{Region II, } E = \frac{\hbar^2 k_2^{\,2}}{2m} + V: \qquad \Psi_{II} = Ce^{k_2 x} + De^{-k_2 x}$$

where the constants A, B, C, D are chosen so that the wave function Ψ and its derivative Ψ' are continuous across the potential boundaries at x = 0 and x = a. Setting $\Psi_I = \Psi_2$ and $\Psi_I' = \Psi_2'$ at x = 0 results in the following two equations:

[II.30] Ψ: $\quad A + B = C + D$

[II.31] Ψ': $\quad ik_1(A - B) = k_2(C - D)$

Similarly, setting the wave functions and their derivatives equal at x = a and using Bloch's Theorem to set $\Psi(a) = \Psi(-b)$ will result in:

[II.32] Ψ: $\quad Ae^{ik_1 a} + Be^{-ik_1 a} = (Ce^{-k_2 b} + De^{k_2 b})e^{ik(a+b)}$

[II.33] Ψ': $\quad ik_1(Ae^{ik_1 a} - Be^{-ik_1 a}) = k_2(Ce^{-k_2 b} - De^{k_2 b})e^{ik(a+b)}$

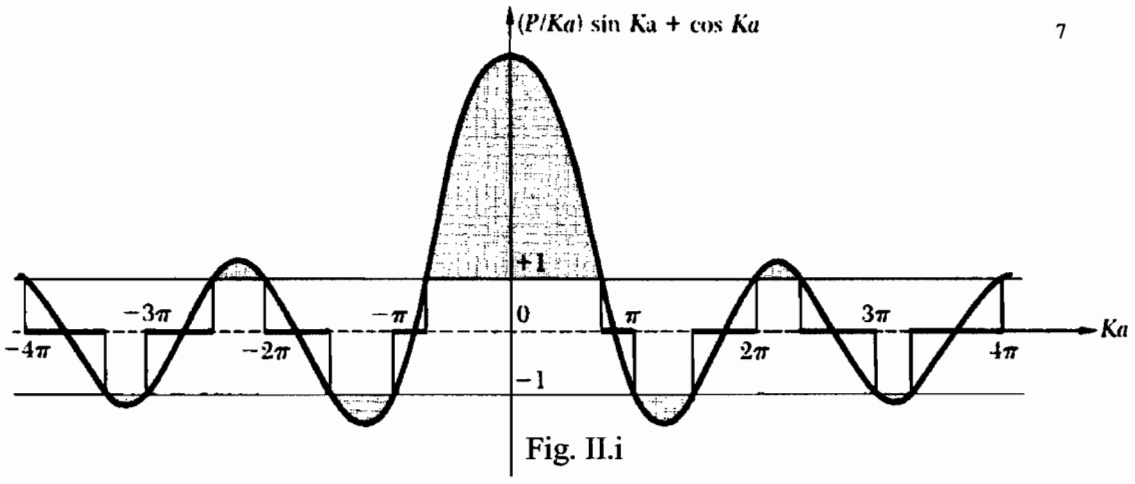
Equations [II.30] – [II.33] can be restated in matrix notation as:

[II.34]
$$\begin{bmatrix} 1 & 1 & -1 & -1 \\ ik_1 & -ik_1 & -k_2 & k_2 \\ e^{ik_1 a} & e^{-ik_1 a} & -e^{-k_2 b}e^{ik(a+b)} & -e^{k_2 b}e^{ik(a+b)} \\ ik_1 e^{ik_1 a} & -ik_1 e^{-ik_1 a} & -k_2 e^{-k_2 b}e^{ik(a+b)} & k_2 e^{k_2 b}e^{ik(a+b)} \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = 0$$

So the equations have non-trivial solutions only if the determinant of the coefficients of A, B, C, D vanishes. After expanding the determinant:

[II.35]
$$\cosh k_2 b \cos k_1 a - \frac{k_1^{\,2} - k_2^{\,2}}{2k_1 k_2} \sinh k_2 b \sin k_1 a = \cos k(a+b)$$

This dispersion relation is extremely complicated, but useful information can be gathered from it by picking proper values for a, b, k_1, and k_2. The most important detail in this result is the right hand side of the equation. Because |cos k(a +b)| is never greater than one, the left hand side of the equation is restricted to stay within that range. Figure II.i plots the left hand side of the equation with b = 0, and the allowed range between ±1. Notice the shaded gaps in the allowed values for k_1, over those ranges, [II.35] has no traveling wave solutions and as a result the gaps in the energy spectrum are formed.



[7]

Fig. II.i

[8]

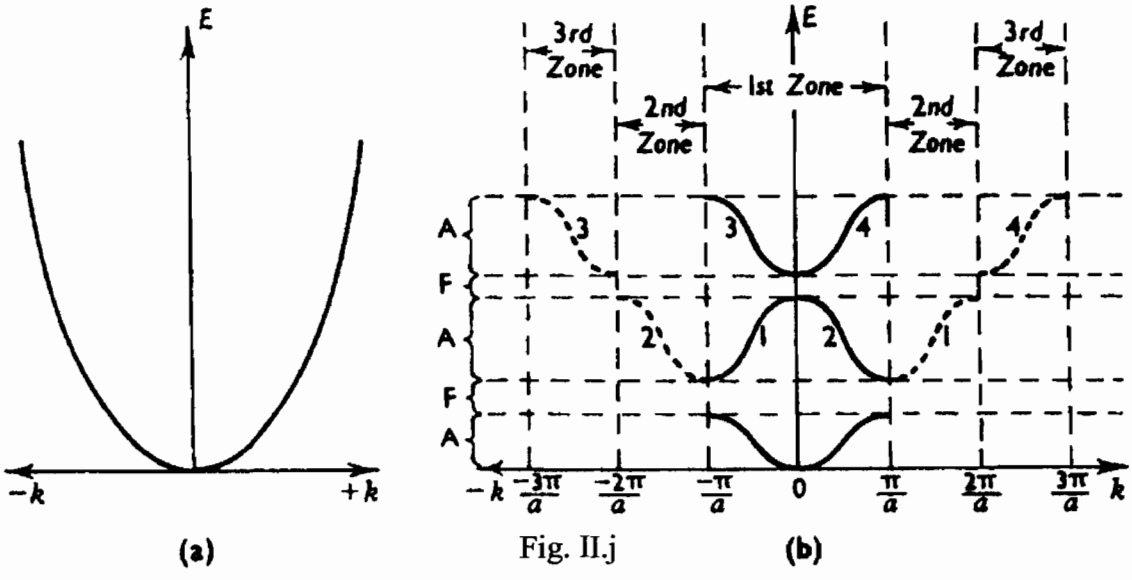


Fig. II.j

A distinct difference can be seen in the plot of energy verses wave vector for both free [II.j.a] and bound electrons [II.j.b]. The free electron model takes on the quadratic shape of [II.16], while the bound electron model has gaps at $\pm n\pi/a$. When n = 1, the electron is in the first Brillouin zone, when n = 2 it is in the second, and so on. The energy gaps govern the behavior of electrons near the Fermi surface and play a crucial role in the properties of conductors, superconductors, semiconductors, and insulators.

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[7] Kittel, Charles

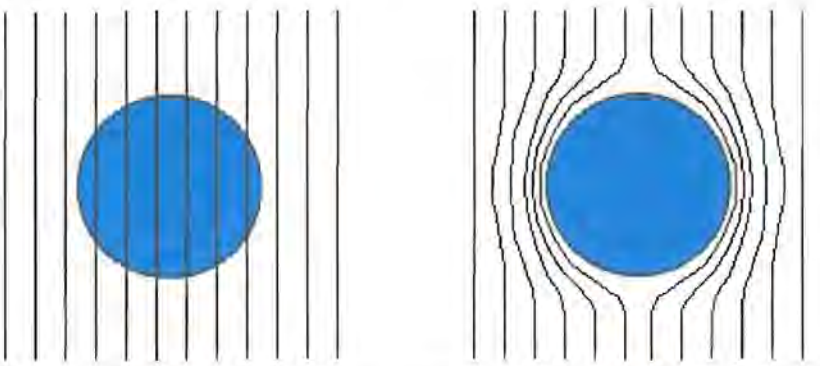
[8] Cusack ,N. The Electrical and Magnetic Properties of Solids. John Wiley & Sons Inc. New York, NY ©1958

Superconductors

Superconductivity is not a bulk property of an object, it is a state of the material that composes an object. Furthermore, a superconducting material is not only characterized by an abrupt loss of electrical resistance, but there are changes in a variety of other properties as well. All these changes occur at the same temperature known as the critical temperature. Some other changes that occur are: the specific heat capacity of the material increases suddenly, thermoelectric effects vanish, and the thermal conductivity of the material changes abruptly. Those characteristics are some of the changes in properties that are observed as the temperature drops below the critical temperature. Because the material changes its properties so dramatically as it passes below the critical temperature, the superconductor is said to pass into the superconducting state.

Soon after Onnes discovered superconductivity, two other interesting properties, critical currents and critical magnetic fields were associated with superconductivity. Obviously, the discovery of resistanceless current flow would cause researchers to run higher and higher currents through superconducting materials. It turns out that once a threshold current is reached (which is usually relatively small), superconductivity is destroyed, even if the superconductor is well below the critical temperature. The current at which that occurs is the critical current and its value depends on the particular material composing the superconductor.

Similarly, superconductivity can be destroyed by a strong enough magnetic field called the critical field. Critical fields are associated with another interesting characteristic of superconductors discovered in 1933 by Walther Meissner and Robert Ochsenfeld. The two experimentalists discovered that when a superconductor is cooled below its critical temperature in the presence of an external magnetic field, then at the transition into the superconducting state, the magnetic flux lines are excluded, or pushed out, from the superconductor and there is no magnetic induction field inside it. This phenomena is known as the Meissner Effect and it is a bulk property of a superconducting specimen.



(a) Fig. II.k (b)

Figure II.k.a shows the material before being brought below its critical temperature and II.k.b shows the magnetic induction lines being expelled from the material. If this effect were to be approached by treating the superconductor as a material with no resistance, then when a material is in the superconducting state, its electrical resistance ρ is zero for any constant current density $\mathbf{J}$. By [II.24] $\mathbf{E} = \rho\mathbf{J}$, so the electric field inside the sample must also be zero. The differential form of Faraday's Law relates the electric field to the magnetic inductance by

[II.36] $$\oint \vec{E} \bullet dl = -\frac{\partial}{\partial t} \iint \vec{B} \bullet \hat{n} dS = -\frac{\partial \vec{B}}{\partial t} A$$

as long as the area through which the magnetic flux is passing does not change over time. So if the electric field is zero, then $\partial \mathbf{B}/\partial t = 0$, which means that the flux through the material cannot change as it cools. This contradicts the Meissner Effect, which shows that the magnetic induction field inside the superconductor must decrease as it is reduced to zero. Therefore, there must be more going on inside the lattice of the material then the electrical resistance dropping to zero. The Meissner Effect suggests that perfect diamagnetism is also an indispensable property of the superconducting state.

Diamagnetism is a form of magnetism that is associated with the change in the orbital magnetic moment of electrons caused by an applied magnetic field. When a magnetic field is applied to a diamagnetic substance, the orbital motion of the electrons in the substance is affected by the field. The changed motion is called a magnetization current. With each magnetization current, there is an associated magnetic dipole moment associated with it. By Faraday's Law, the dipoles will be induced in such a way so that they will oppose the field that induced them. The sum of all those dipoles is equal to a quantity called the magnetization. From classical electromagnetism, the magnetic inductance B is related to the magnetic field H and the magnetization M through:

[II.37] $$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$$

The ratio of the magnetization to the magnetic field is a dimensionless quantity χ called the magnetic susceptibility ($M/H = \chi$). For diamagnetic materials, χ is a negative quantity. For a perfect diamagnet, $\chi = -1$, which means $M = -H$ and [II.37] reduces to $B = 0$, the result demonstrated by the Meissner Effect. When the magnetic field surrounding the superconductor is strong enough, the magnetization drops to zero and the magnetic field penetrates the material, destroying the superconducting state. The critical field at which that happens is denoted by H_c and its strength varies from material to material. Some materials, called type I superconductors, experience sharp cutoffs in magnetization while other materials, called type II superconductors experience a decrease in magnetization after a critical field H_{c1} is reached. They continue to decrease in magnetization as the magnetic field is increased until a second critical field H_{c2} where the magnetization goes to zero. Figure II.k shows the magnetization curves for type I and type I superconductors. For a given critical temperature, the area under the magnetization curve is the same for a type I superconductor as it is for a type II[9]. Type II superconductors in an external magnetic field between H_{c1} and H_{c2} are said to be in a vortex state because there are pockets of matter in the normal state surrounded by superconducting regions and the magnetization currents flow around the normal regions like a vortex of a tornado. The magnetic field surrounding the sample will penetrate the normal regions and creates a magnetic flux through a region near the surface of the material. Because the magnetic field cannot penetrate all the way through the sample, a type II superconductor will remain in a mixed "partially" superconducting state until the second critical field is reached. Type I superconductors are generally the elements on the

[9] Kittel

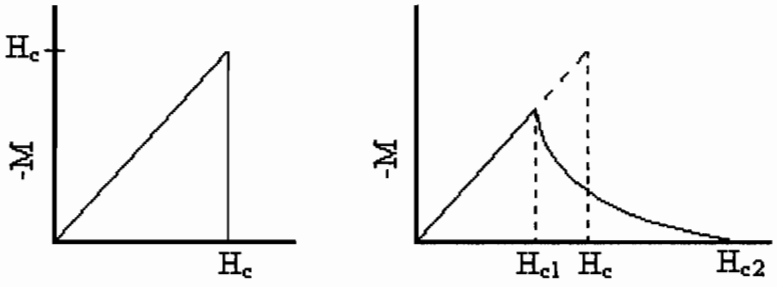


Fig. II.k

periodic table that are known superconductors, i.e. pure metals and transition metals, while type II superconductors are usually alloys and compounds.

By now it should be obvious that superconductivity is closely tied to both zero resistance and the Meissner Effect, two phenomena that are both dependent on the behavior of electrons and hence related to the Fermi surface. Remember that the Fermi Surface is not always at a constant level due to quantum and thermal excitations in an atom. Electrons near the surface could absorb a photon, or be excited with the application of heat causing the electrons to momentarily "jump" above the Fermi surface. If the surface happens to be at the same level as a band gap, then the atom's outermost energy band is completely filled. In this case, an electron needs to gain a large amount of energy to jump to the next level above the band gap. If the Fermi surface lies well inside an energy band then electrons have no problems gaining energy, and thus have an easy time of traveling from atom to atom. Conductors are materials that contain atoms that

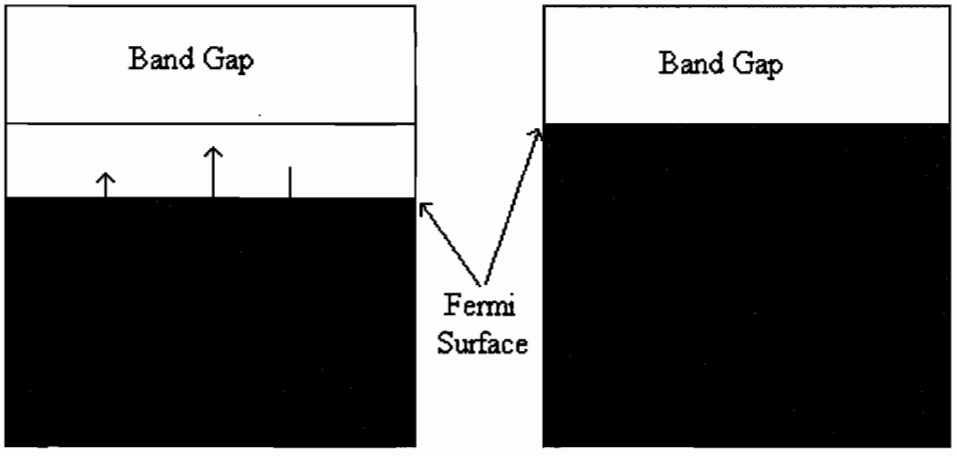


Fig. II.l

have partially filled energy bands (Fig. II.k.a), and the electrons near the Fermi surface are called conduction electrons, because they are the easiest to excite and remove from their atoms to generate a current. An insulator is a material that has its energy band completely filled (Fig. II.k.b), which makes it hard for electrons in such a substance to gain enough energy to jump to the next energy level and become a conduction electron. The superconducting state does not just involve electron-lattice interactions (that generate the energy bands), but as a result of the electrons traveling through the lattice, involves electron-electron interactions as well.

In 1957, the physicists Bardeen, Cooper, and Schrieffer proposed an atomic theory of superconductivity, called BCS Theory in their honor, based on electron-electron interactions within the lattice of a solid. BCS Theory is based on the strange concept of an attraction between electrons, caused by lattice vibrations. At first thought an attraction between electrons seems impossible, because electrons contain negative charges and thus should repel one another through Coulomb interactions. To understand how two electrons could be attracted to one another, consider the oversimplified case of two static electrons within a lattice. Surrounding each electron are the positive ion cores of the lattice. The cores are not held rigidly in place, but they vibrate as if connected by springs. The negatively charged (exaggerated) electrons attract all of the nearby positive cores as in Fig. II.m. The attraction of the cores by one electron results in a concentration of positive charge around that electron. In other words, the lattice is polarized by the electron. The concentration of cores results in a net positive region of the lattice that attracts the second electron. The process works so that both electrons mutually attract one another, although the force is not great enough to overcome the repulsion due to the charges of the electrons.

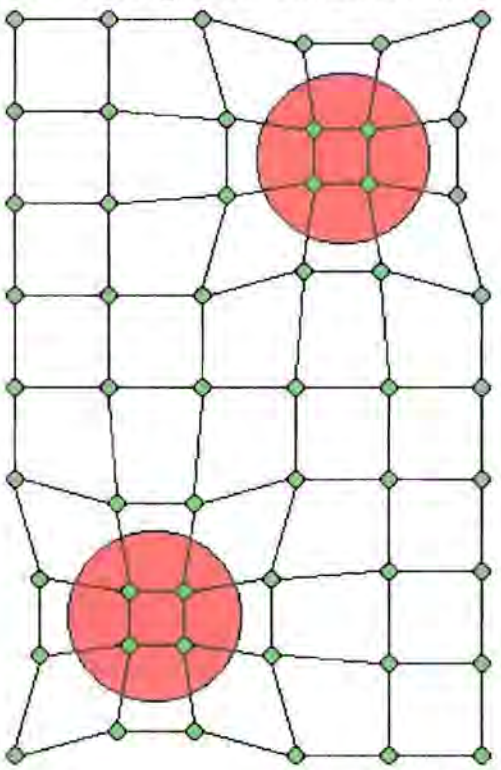


Fig. II.m

The situation can be described using an analog similar to the way gravity is described in general relativity. The electrons behave like two balls and the lattice like a thin elastic membrane. When the balls are placed on the sheet, their weight deforms the membrane. This deformation represents the polarization of the lattice due to the electron's charge. The energy of the whole system can then be reduced by having the balls sink towards each other into a single trough, so to reduce their potential energy the balls become in a sense bonded to one another. Of course, the Coulomb force prevents the electrons from actually touching, but the reduction in potential energy of the system has gone into the formation of a bond between the two electrons.

The flaw in the description above is that electrons in the lattice are moving with considerable velocities and do not polarize the lattice statically. Instead, the polarization is set up along the path that the electron travels. Because the lattice is an able to vibrate, the amount of polarization will depend on the natural frequencies of vibration that the lattice can have. That would indicate that the size of the cores would have an effect on superconductivity. Heavy atomic isotopes have lower lattice frequencies and are slower to concentrate, thus the attraction between electrons would be less. Therefore, the electron-electron interaction is weaker, corresponding to a lower critical temperature. Lower critical temperatures in heavy isotopes have indeed been observed experimentally

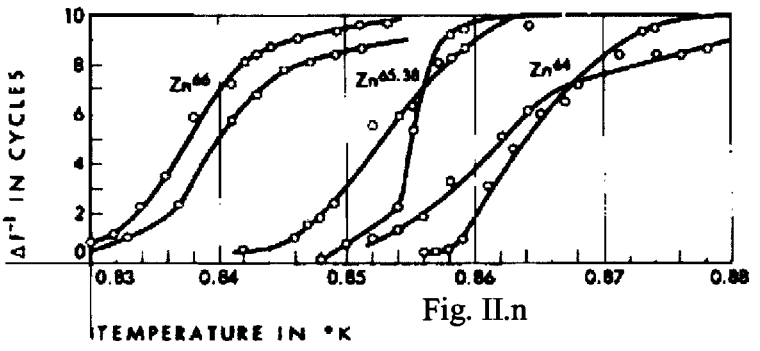


Fig. II.n

[10] as demonstrated for zinc in Figure II.m. Each line on the graph represents the transition between the normal state and the \ superconducting state. The heavier the isotope, the lower the temperature at which the transition occurs.

If electrons are moving through the lattice, then how do two electrons become attracted? The new dynamic model now requires that the second electron follow the polarization track of the first electron. That will reduce the energy of the electron because the lattice will already be in a polarized state. There are two ways that the electrons can follow the same path: The first is they can have the same momentum ($\mathbf{p}_1 = \mathbf{p}_2$), in which case we can easily consider the two electrons a single particle with a total momentum of twice that of a single electron. The second, and more abstract option, is that the electrons are traveling along the same path, but they have opposite momentum ($\mathbf{p}_1 = -\mathbf{p}_2$), and can be considered a single pair with a total momentum of zero. Again, the Pauli Exclusion Principle plays a crucial role by not allowing two electrons to have the same momentum and excluding the first case for electron propagation throughout the lattice. It also turns out that it is energetically favorable for the electrons to have opposing spins. An electron pair with opposite momentum and opposite spin is called a Cooper pair, because it was Cooper who first discovered that such a pairing of electrons would lead to a reduction in energy of the pair. Fermions are particles with half integer spins, a classification that electrons fall into because they can have a spin of ±½. Treating a Cooper pair as a single particle, its spin can be determined by summing the spins of its constituent particles. Thus, a Cooper pair made up of two electrons with spin +½ and -½ respectively will have a spin of zero. Particles with integer numbers of spin are called Bosons, so obviously, a Cooper pair is a Boson. Bosons do not obey Fermi-Dirac Statistics, and thus do not have to obey the Pauli Exclusion Principle. Thus, every Cooper pair in the lattice can have the same momentum, which due to the opposite momentum of the electrons in the pair, is zero in the ground state. However, if a potential difference is applied across the lattice, then every Cooper pair in the lattice is accelerated and they all gain momentum.

Obviously, a particle with momentum p and energy $p^2/2m$ can enter into collisions with the lattice. After such a collision with the lattice the particle has lost momentum q and energy $q^2/2m$. Then the momentum of the particle is changed to $p - q$ and its energy to $(p-q)^2/2m$. The energy of the whole lattice-particle system needs to be conserved so:

[II.38]
$$\frac{p^2}{2m} - \frac{(p-q)^2}{2m} = \text{Energy lost to the lattice}$$

[10] Geballe, T. H. & Matthias, B. T. (1962) Isotope Effects In Low Temperature Superconductors. *IBM Journal of Research and Development.* Volume 6(2). 256 – 257.

The dispersion relation $\omega = kv_p$ helps to quantitatively determine the lost energy. When a particle hits the lattice, it sets up a vibrational wave that travels through the lattice. In quantum mechanics, particles can be described as waves and vice versa. The particle counterpart to a vibrational wave in a lattice is called a phonon. When the particle hits the lattice the phonon is released with energy $\hbar\omega$, where ω is the angular frequency of the vibrational wave. The dispersion relation connects the angular frequency to the wave's speed of propagation through a material, which in this case is the speed of sound in the material. Thus, the energy of the phonon is $\hbar kv_p$, where k is the wave number of the wave. The momentum of the phonon is equal to the momentum lost to the particle so that $q = \hbar k$. The energy of the phonon can now be written as $E = v_p q$, and [II.38] becomes:

[II.39] $$\frac{p^2}{2m} - \frac{(p-q)^2}{2m} = v_p q$$

this can be reduced to:

[II.40] $$v = \frac{p}{m} = v_p + \frac{q}{2m}$$

Because the electron *loses* momentum to the lattice, q is a positive quantity, so according to [II.40] $v > v_p$. The particle will only lose energy to the lattice if the speed of the particle is greater than the speed of propagation of a phonon in the lattice. Sound waves are vibrational waves, so it is correct to say that if the velocity of the particle is less then the speed of sound in the material, then the particle will not interact with the lattice. That could be stated quantum mechanically by saying that the particle does not have enough energy to interact with the lattice and give up a "packet" of energy. Suppose that the particle is an electron. If the electron is traveling below the speed of sound in the lattice, then it will conduct without colliding with the cores of the lattice. Unfortunately, only about 0.000001% of electrons in the normal state actually travel with velocities that low[11] because the electrons cannot all have the same momentum, resulting in a measurable resistance. In the superconducting state, Cooper pairs can hold the same momentum, and in fact, they all have the same momentum. As long as their velocities remain below the speed of sound of the lattice, the pairs will travel unhindered through the crystal and a resistanceless current ensues.

Before the Cooper pairs form within the lattice, the material is not in the superconducting state. As the material is cooled, the Cooper pairs begin to form and because they are bosons, they all fill one energy state, which in the absence of an electric field will be the lowest possible state. This "condensation" of the Cooper pairs leaves an energy gap, the magnitude of which is equal to twice the energy difference between the superconducting ground state and the normal Fermi surface. If that energy difference is Δ, then the energy gap has a magnitude of 2Δ. Electrons and Cooper pairs cannot occupy the energy gap, so the energy required to excite the system is also 2Δ. The size of the energy gap will decrease as the superconductor warms, so that when it is at the critical temperature, $\Delta = 0$. It also has a limiting size, which is when the superconductor is at absolute zero. In that case, all of the Cooper pairs will have condensed into the lowest

[11] Taylor, A. W. B. Superconductivity. Wykeham Publications. London England. 1970. Pg 81.

possible ground state, leaving the largest possible energy difference between the ground state and the Fermi surface of the normal state.

The energy gap can be measured experimentally measuring the absorption of photons by the material. If the superconductor is formed into a cavity, and photons are directed into the cavity, as long as the energy of the photons, $h\nu \leq 2\Delta$, then the Cooper pairs will not absorb them, and they will be deflected. When the photons have enough energy, the Cooper pairs will absorb them and be excited above the energy gap, resulting in a decrease in the number of reflected photons.

If the Cooper pairs are brought above the energy gap, then they have enough energy to break the "bonds" that form the pair. Placing a potential difference across the superconductor will increase the energy of all the Cooper pairs. As the current increases, so does the velocity of the Cooper pairs, and when a threshold velocity is reached, the pairs have enough energy to overcome the energy gap and break apart into individual electrons, destroying the superconducting state. The critical current has been explained! It turns out that the energy required to break the Cooper pairs is on the order of 10^4 less than the energy required for the pair to reach the speed of sound in any lattice, so that the Cooper pairs will always satisfy the requirement that $v < v_p$.

The Meissner effect can also be explained by Cooper pairs. Remember that when a static magnetic field is applied to a diamagnetic material, the freely moving electrons are forced to move in such a way that the magnetic field they produce is opposed to the applied magnetic field. One would think that the magnetic moments of all the electrons would add together to form a strong magnetic opposing the applied field. However, the Pauli exclusion principle forbids the electrons to have the same momentum, and the overall effect is weakened because the magnetic moments of the electrons are not aligned in the same direction. In the superconducting state, the Cooper pairs (in the absence of any electric currents) all have a net momentum of zero. When the magnetic field is applied, the Cooper pairs are all forced to move in the same direction and their magnetic moments all line up exactly with one another. The result is perfect diamagnetism exhibited by the Meissner effect. The energy from the applied field is imparted to the Cooper pairs, however if that energy is great enough, then the "bonds" that hold the pairs together will be broken. In other words, a critical field breaks the Cooper pairs apart, resulting in resistance causing electrons and a reversion into the normal state.

Another way of supplying energy to Cooper pairs to break them apart is through heat. Thermally exciting the pairs will eventually cause them to break apart once a particular temperature is reached. In other words, the critical temperature of the superconductor is dependent on the energy it takes to form the Cooper pairs inside the lattice.

BCS theory of superconductivity as described above predicts many interesting phenomena and has been verified by experimental facts, but it still fails to fully explain the behavior of type II superconductors. BCS theory can be used as a reasonable approximation, but the actual details of the physics of type II materials are still being worked out. However, BCS theory covers a significantly broad range of topics and describes enough physical results to understand the processes and procedures involved in the construction of a device that measures critical temperatures of homemade superconductors made.

SECTION III: Setup & Experiment

Critical Temperature Measurement Device

The premise of the experiment was to use an electronic signal to measure the Meissner Effect by passing it through an amplitude detector and a low-pass filter. Two solenoids, a primary and a secondary, were constructed using PVC piping and insulated copper wire. To build the primary solenoid, 500 coils of 0.94mm wire was wrapped around 2-inch diameter PVC pipe and coated with Liquitex Gloss Gel Medium to hold the wire permanently in place. Two grooves were cut into the PVC pipe at the top of the primary so that a support for the secondary solenoid could lie across the diameter of the primary. The primary was then mounted on a circular wooden base, around a wooden pedestal that penetrated 1-inch into the bottom of the primary. The secondary solenoid was built around ¾-inch PVC pipe and constructed using two 750-coil sections of 0.31mm wire. Each section started two inches from the center of the pipe (one above and one below center) and consisted of a 250 coil base wrap that was coated with the Liquitex Gloss Gel Medium. After the 250th coil, the wire was run straight down the coil, 250 more turns were added to make a second coil on top of the base coil, and more Gloss Gel added. The wire was again run straight down the coil on the opposite side of the first run. The final 250 turns made a third coil and a last layer of Gloss Gel added. One inch was cut off the bottom of the secondary to account for the wooden pedestal used to mount the primary. A ¾-inch diameter styrofoam cylinder was installed so that its top sat at the midpoint of the top coil on the secondary, and Liquitex Gloss Gel was applied to the styrofoam to hold it in place. That coil sits at the top of the device and was named the experiment coil because it holds the superconductor. The bottom coil was called the reference coil because it is used as a control. A styrofoam toroid of outer radius 2-inches and inner radius ¾-inches was placed at the base of the secondary to help stabilize it.

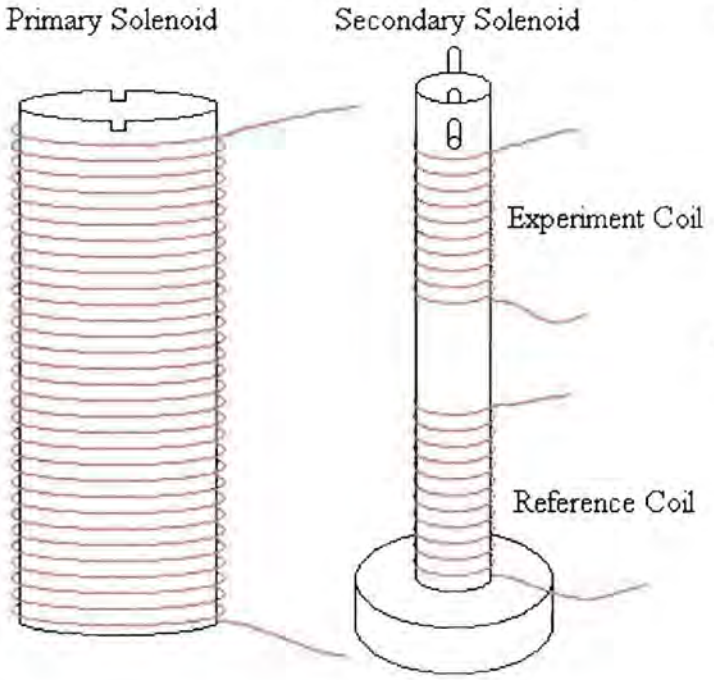


Fig. III.a

Two holes were drilled in the top for a plastic support rod that would sit in the grooves cut into the primary. The wires from both the primary and secondary solenoids were soldered to BNC adapter leads.

The device was then connected to a Teachspin SPLIA1-A Signal Processor using coaxial cables with BNC adaptors. The signal generator in the SPLIA1-A was fed directly to the primary solenoid, while the experiment coil in the secondary solenoid was connected to the A jack on the preamplifier and the reference coil was connected to the -B jack. The preamplifier gain was set to 50V and its output was fed into the amplitude detector on the SPLIA1-A, and from the detector, into the low-pass filter/amplifier that was also built into the device. Both the detector and the low-pass filter/amplifier gain were set to 2V, the time constant dial and the roll off switch on the low-pass filter/amplifier were set to 0.1s and 12 dB/oct. respectively. A wire schematic of this part of the setup is included as Figure III.b. Then the signal passed to a Pasco Science Workshop 500 Interface. Because the leads into the data port require banana plugs, a BNC to banana coaxial cable connected the output of the low-pass filter/amplifier to the input for the Pasco 500. The Pasco interface was then connected to a computer so data would be taken using Pasco's Data Studio data collection program.

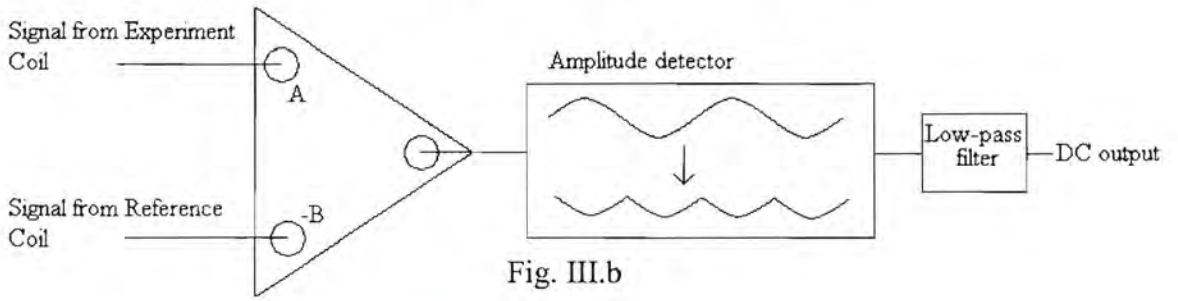


Fig. III.b

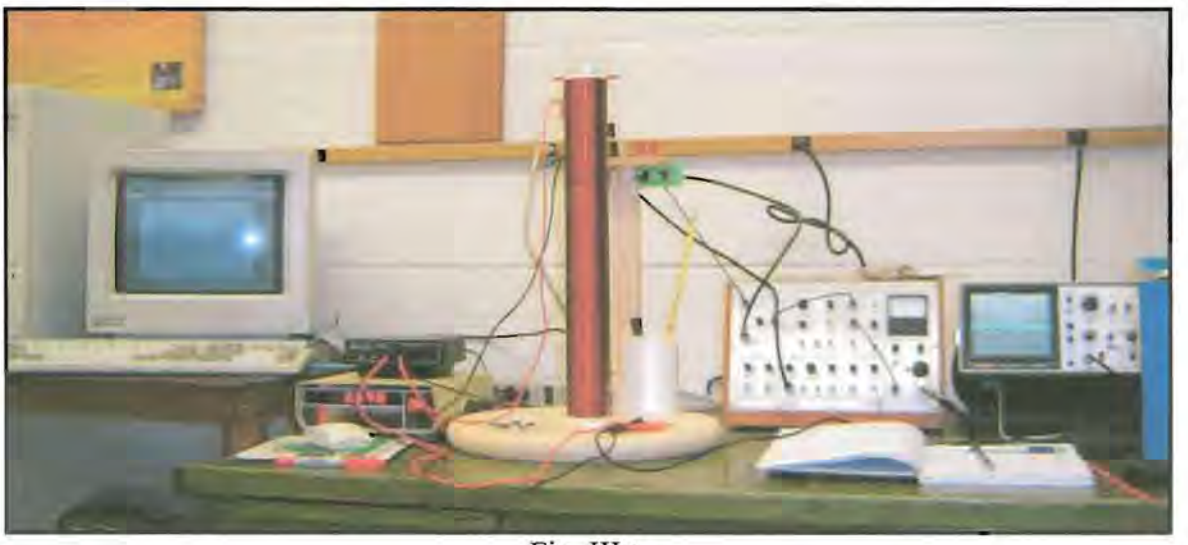


Fig. III.c

Thermocouple

The thermocouple used in the experiment was an Omega Engineering copper-constantan fine wire thermocouple. It was made out of copper and constantan, two metals with different thermal properties. When the metals change temperature, they expand, and a potential difference is set up across the junction between them. The experiment requires two such junctions. One kept at a reference temperature, so it was possible to determine the temperature of the second junction by measuring the voltage generated in the thermocouple. After soldering the constantan parts of two junctions together, the copper leads were tied to banana plugs. Banana plug wires then connected the thermocouple to a Keithley 177 Microvolt Digital Multimeter that was set to display the voltage in the thermocouple at the 20mV setting. The Keithley DMM also had a built-in amplifier with a 100V gain, the output of which was fed through banana plugs into the Pasco 500 so Data Studio could collect the data coming from the thermocouple. The reference junction was sealed in a test tube filled with mercury to ensure good thermal contact of the junction. The test tube was then placed in an ice bath so the reference temperature could be maintained constant at 273 K, the freezing point of water. The other junction was taped to the sample that was being tested in the experiment. To zero the thermocouple, both junctions were placed in mercury-filled test tubes and the tubes were placed in ice water. A voltage could be read using the Keithley DMM, which was then zeroed so that a temperature at the sample junction of 273 K would correspond to a reading on the DMM of 0V. A calibration curve for the thermocouple was used to convert the voltages into temperatures was obtained from Omega Engineering and has been included in Appendix C.

Disks

There were six disks that were studied in using the critical temperature measurement device, each was assigned a label from 1 – 6. The first three disks were produced by Colorado Superconductors, a science supply company that specializes in making superconducting disks. It is believed that those disks were made out of $Bi_2Sr_2CaCu_2O_9$. Disks 4 – 6 were produced from $YBa_2Cu_3O_7$ kits[12] by students enrolled in Lycoming College's Modern Physics course.

[12] Colorado Superconductor Inc. Instruction Manual for Superconductor Demonstrations. Fort Collins, Colorado. ©1987

The Experiment

The secondary solenoid sits as close to the center of the primary solenoid as possible. The signal generator in the detector generates an alternating current that passes through the primary solenoid, and sets up an oscillating magnetic field inside the primary. The time-varying magnetic field then induces alternating currents in the secondary coils, which are fed into the preamplifier in the SPLIA1-A. The signal from the reference coil is inverted by the preamplifier, and then mixed with the signal from the experiment coil. When the experiment coil has no samples in it, its signal will be the same as in the reference coil and the two signals will cancel with one another when mixed by the preamplifier. However, if a magnetic material is placed in the experiment coil, then the magnetic field inside that coil will be distorted. The distortion of the magnetic field induces another current in the experiment coil that adds to the one already in it, and there is a difference between the signals generated in the secondary solenoid coils. Thus, when the preamplifier mixes those two signals, there will be a resulting overall AC signal. The combined signal is sent to the amplitude detector where it is negatively rectified, amplified, and then passed on to the low-pass filter/amplifier. The low-pass filter averages the rectified signal, turning it into a DC voltage, and a built in amplifier will step up the voltage so that the Pasco 500 can read it. The purpose behind rectifying the signal is now apparent; if the signal were a plain AC signal, the low-pass filter would always average it to zero. Now the only way for the Pasco 500 to read a zero DC voltage is if the signals that pass into the preamplifier are the same.

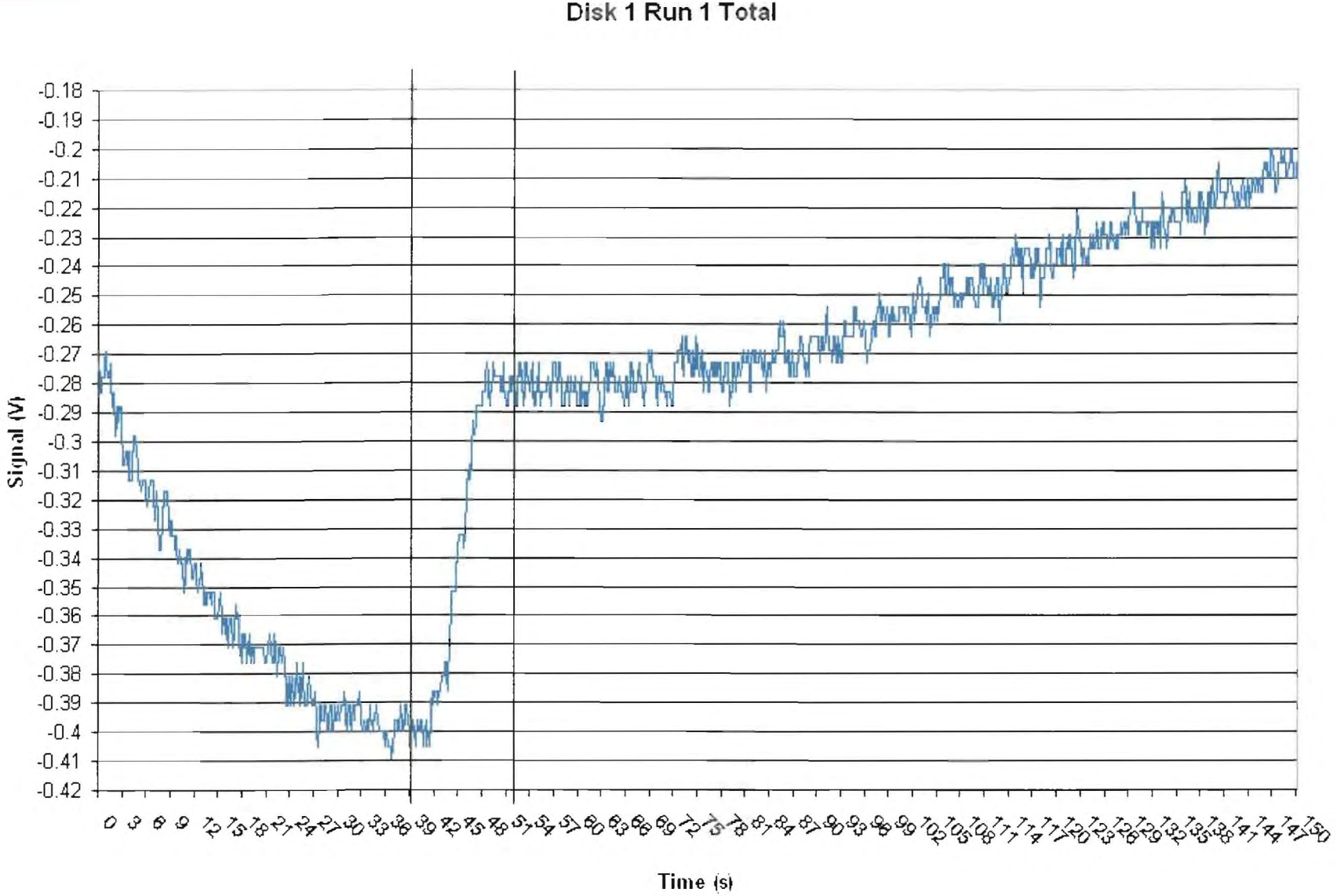
When a superconductor is cooled below its critical temperature, it acts as if it were a perfect diamagnet. Thus, when a superconductor is in the superconducting state, the Meissner effect distorts magnetic fields. That fact allows the detection of the transition between the superconducting state and the normal state. When cooled with liquid nitrogen, the superconductor will distort the magnetic field inside the experiment coil, and a new current will be induced in that coil. The difference will ultimately mean a negative, nonzero DC output from the low-pass filter/amplifier.

As the superconductor warms, it will pass through its transition temperature. The signals between the experiment and reference coils will again become the same, and the DC output will go to zero. It is easy to determine the critical temperature of the superconductor by measuring the temperature of the superconductor and the DC output from the low-pass filter/amplifier as functions of time. The transition is obviously not an instant drop in the DC voltage to zero, because the outside of the superconductor passes the critical temperature before the inside, thus there is a period of time over which the signal decay occurs. If the shape of the sample were spherical, then the transition would be linear because the sphere would heat up at the same rate in all directions. Disk shaped samples lose heat faster from the top surfaces and slower at the edges, resulting in an almost linear transition that is curved at the beginning and the end.

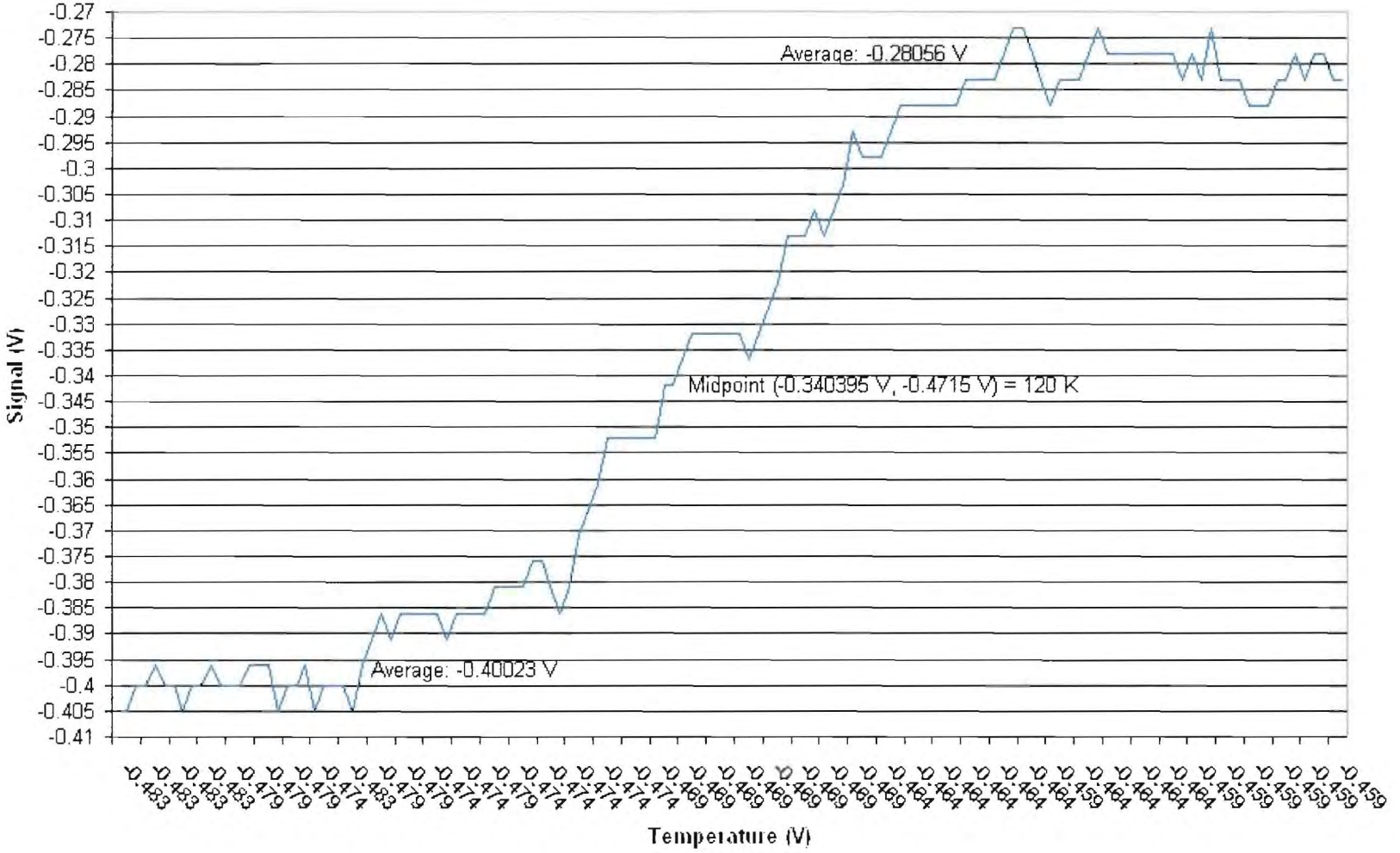
In this work, the critical temperature is the considered to be the temperature at the midpoint of the transition. The time that corresponds to the midpoint of the DC signal voltage also corresponds to a voltage output from the thermocouple. Thus, the calibration curve can convert the thermocouple output into a temperature to resolve the critical temperature of the material.

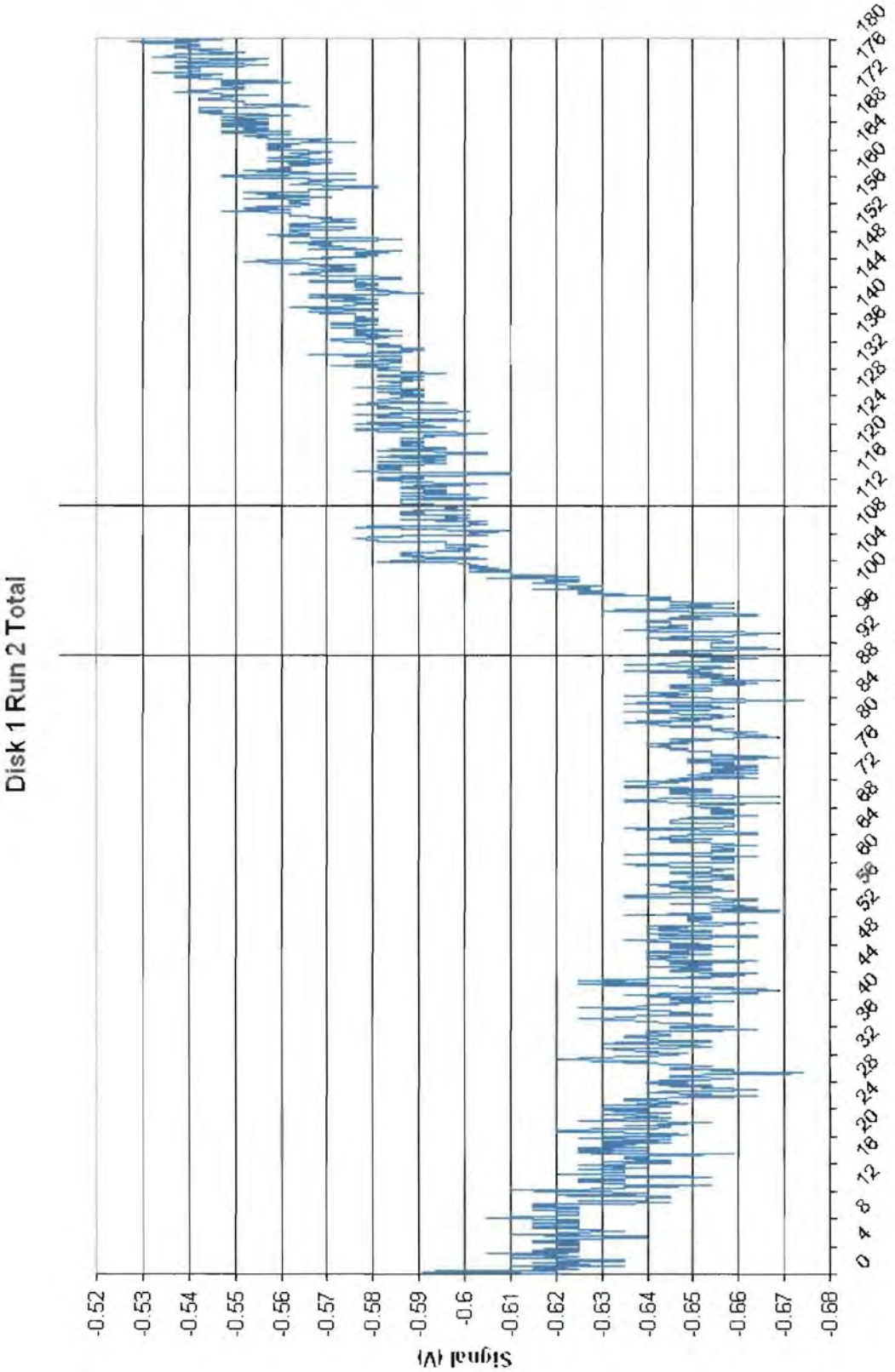
CTION IV: Results

The next section contains data taken for two of the six disks that were tested. Not of the data was included because the disks that were chosen demonstrated behavior licative of the rest of the disks. Disk 1 is representative of the disks composed of $_2Sr_2CaCu_2O_9$ (disks 1 – 3). Disk 4 is representative of the $YBa_2Cu_3O_7$ disks (4 – 6). ch set of data consisted of five different data runs, and each run has two graphs sociated with it. The first graph is a plot of the whole data run and is labeled with otal" in the title. This plot allowed the time period over which the transition occurred be determined and is marked by two horizontal lines on each total run graph. The cond graph for each run is a magnification of the time period determined from the first aph. The critical temperature was determined from the second graph by averaging the nal before and after the transition and calculating the midpoint of the two averages. e temperature of the thermocouple at the midpoint signal was taken to be the critical mperature of the device. In instances where there were multiple values of the signal ltage at the midpoint and different corresponding temperatures, the median of the mperature range was taken to be the critical temperature. Data for disk 1 can be found pages 30 – 39. There were two sets of data for Disk 4, labeled 4 and 4b, these can be nd on pages 40 – 63. Two plots of the signal noise of the device can be found on ges 64 – 67, one plot of the noise for 1500s and another plot of the noise vs. mperature of the inside of the solenoid. The last two graphs of this section are plots of ta taken using Lycoming College's four-point probe method for comparison with ults from the critical temperature measurement device.


Disk 1 Run 1 Total
Signal (V)
-0.18
-0.19
-0.2
-0.21
-0.22
-0.23
-0.24
-0.25
-0.26
-0.27
-0.28
-0.29
-0.3
-0.31
-0.32
-0.33
-0.34
-0.35
-0.36
-0.37
-0.38
-0.39
-0.4
-0.41
-0.42
Time (s)

Disk 1 Run 1


Average: -0.28056 V
Midpoint (-0.340395 V, -0.4715 V) = 120 K
Average: -0.40023 V
Signal (V)
Temperature (V)
-0.27
-0.275
-0.28
-0.285
-0.29
-0.295
-0.3
-0.305
-0.31
-0.315
-0.32
-0.325
-0.33
-0.335
-0.34
-0.345
-0.35
-0.355
-0.36
-0.365
-0.37
-0.375
-0.38
-0.385
-0.39
-0.395
-0.4
-0.405
-0.41
0.483
0.479
0.474
0.469
0.464
0.459


Disk 1 Run 2 Total
Signal (V)
-0.52
-0.53
-0.54
-0.55
-0.56
-0.57
-0.58
-0.59
-0.6
-0.61
-0.62
-0.63
-0.64
-0.65
-0.66
-0.67
-0.68
Time (s)
0
4
8
12
16
20
24
28
32
36
40
44
48
52
56
60
64
68
72
76
80
84
88
92
96
100
104
108
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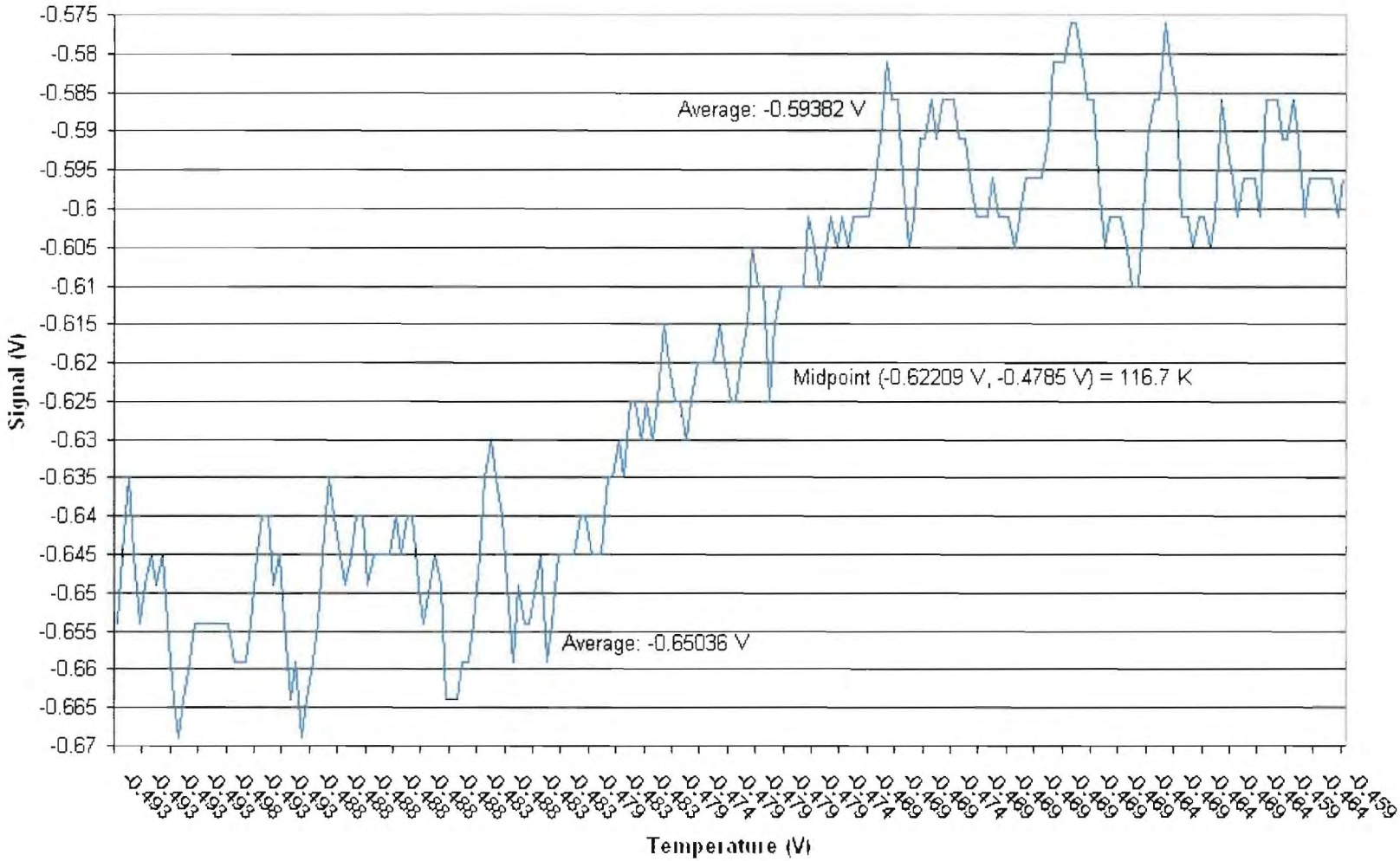
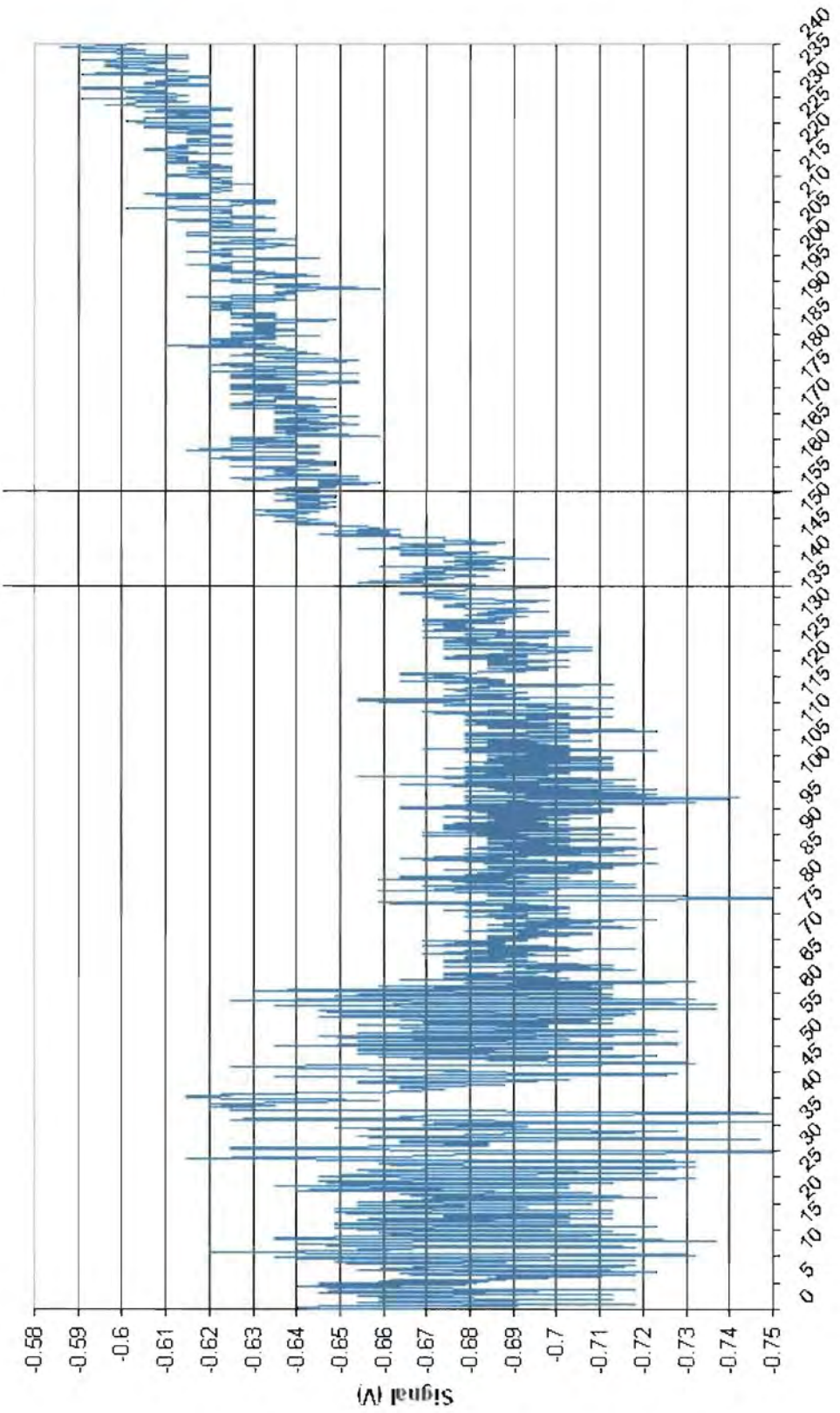
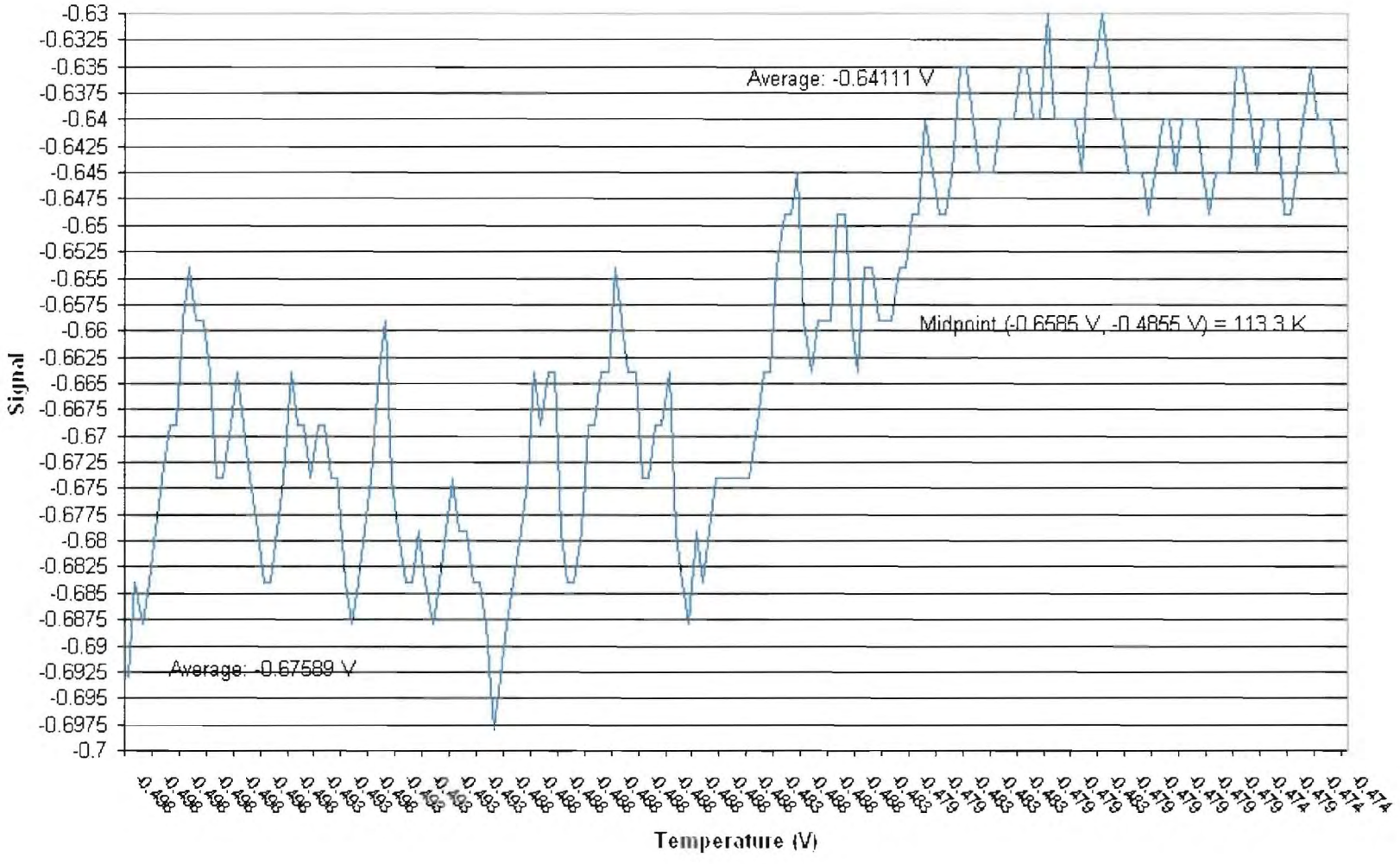
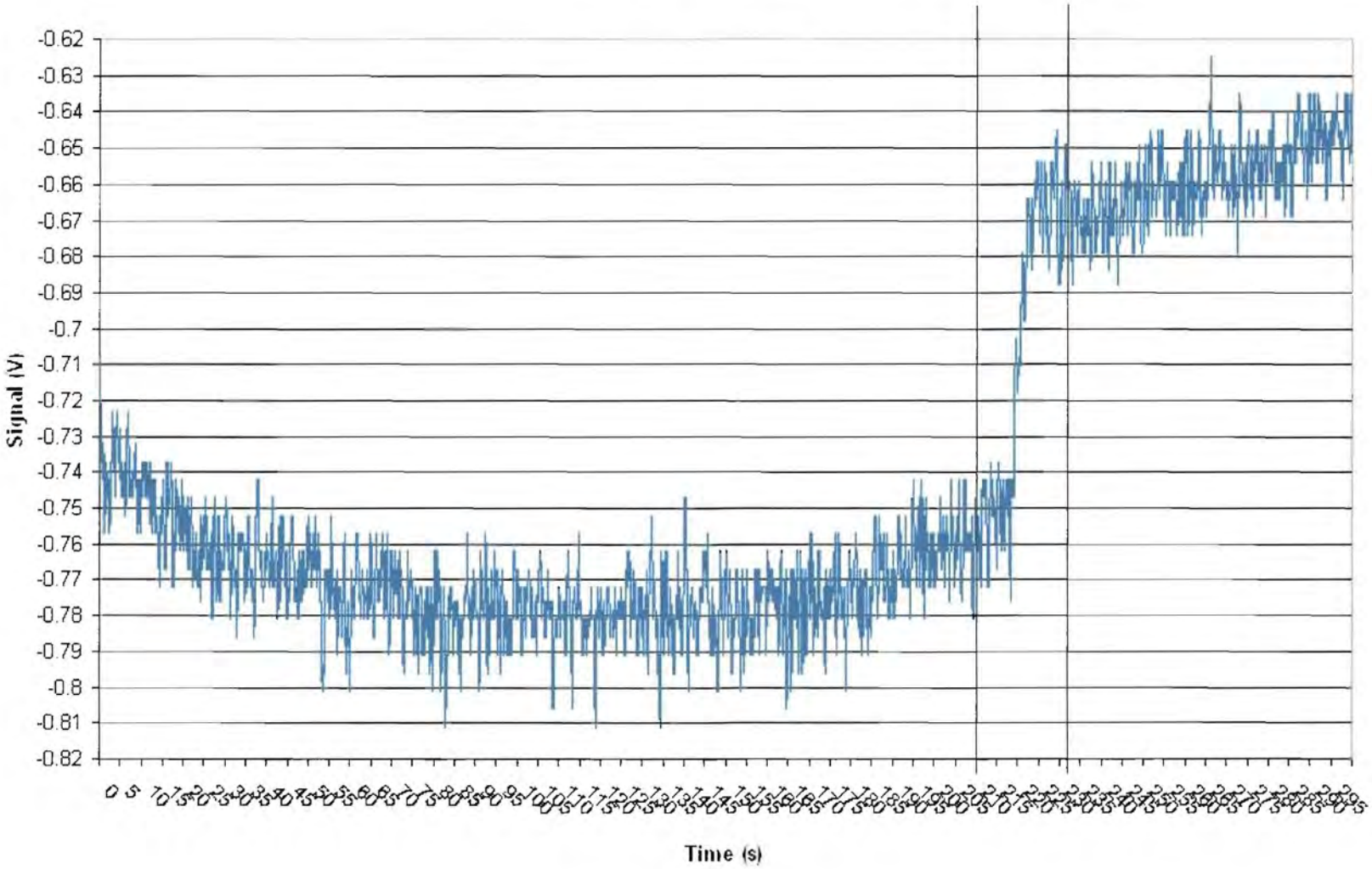
128
132
136
140
144
148
152
156
160
164
168
172
176
180

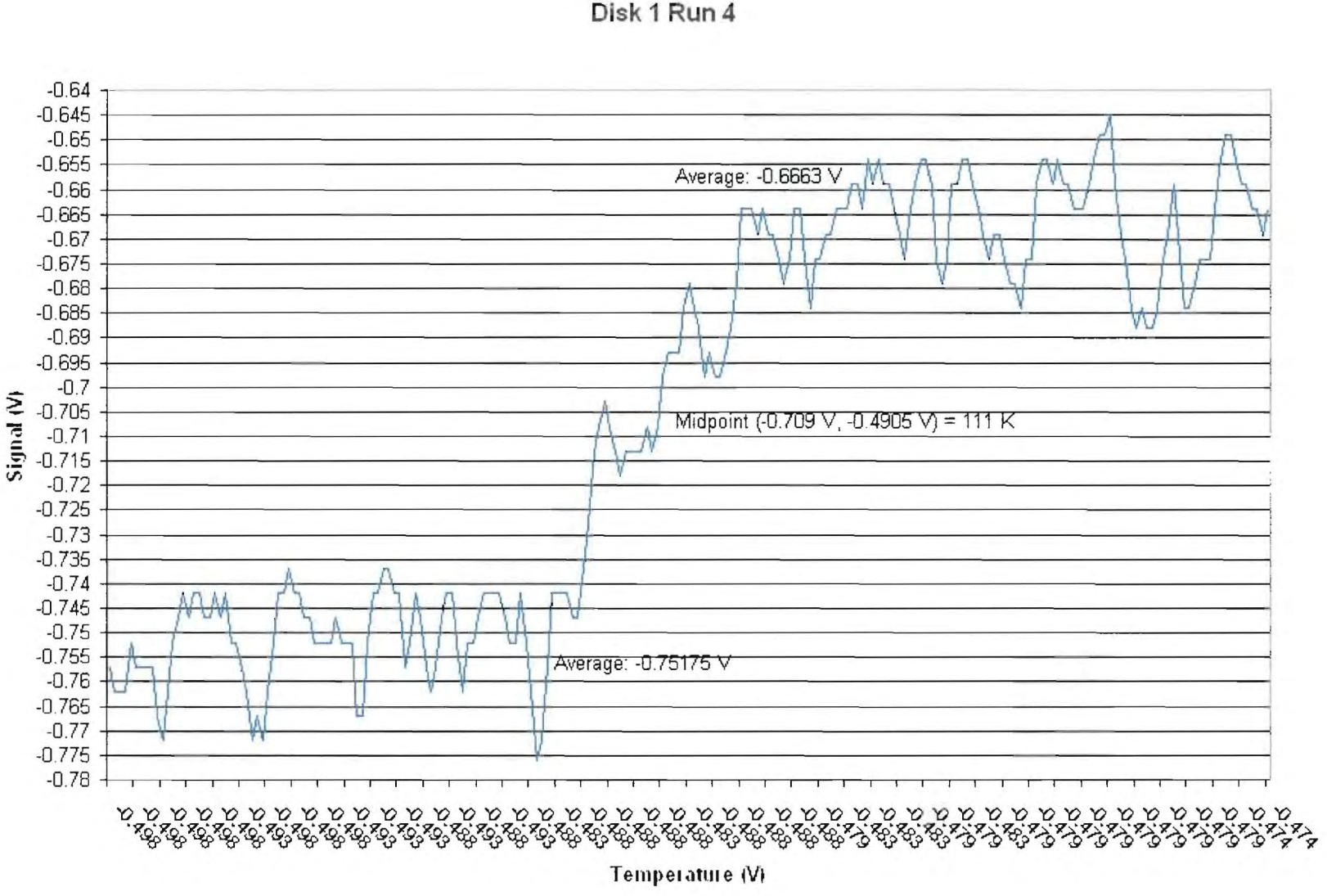

Disk 1 Run 2
Average: -0.59382 V
Midpoint (-0.62209 V, -0.4785 V) = 116.7 K
Average: -0.65036 V
Signal (V)
Temperature (V)
-0.575
-0.58
-0.585
-0.59
-0.595
-0.6
-0.605
-0.61
-0.615
-0.62
-0.625
-0.63
-0.635
-0.64
-0.645
-0.65
-0.655
-0.66
-0.665
-0.67

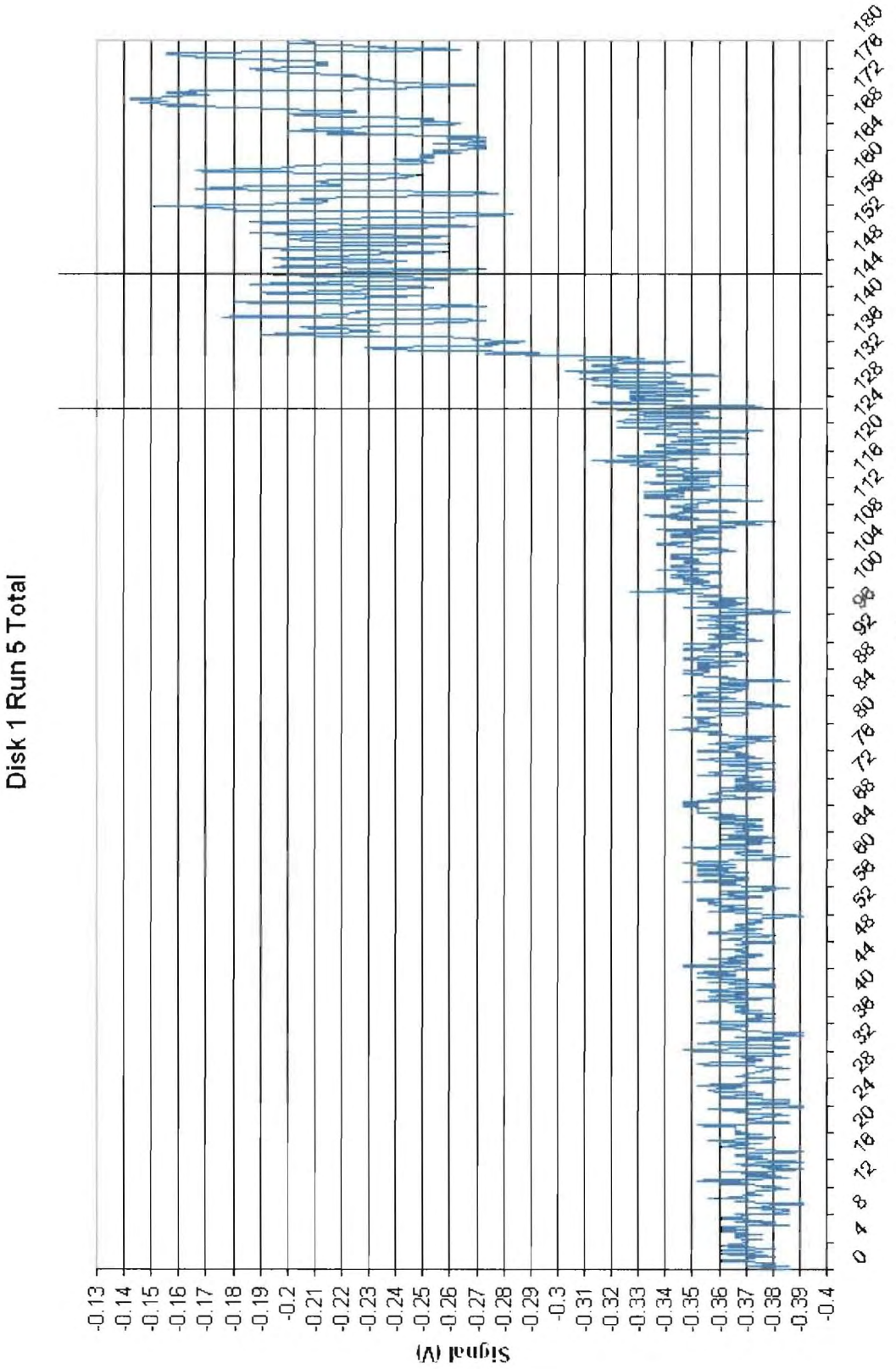

Disk 1 Run 3 Total
Signal (V)
-0.58
-0.59
-0.6
-0.61
-0.62
-0.63
-0.64
-0.65
-0.66
-0.67
-0.68
-0.69
-0.7
-0.71
-0.72
-0.73
-0.74
-0.75
Time (s)
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140 145 150 155 160 165 170 175 180 185 190 195 200 205 210 215 220 225 230 235 240

Disk 1 Run 3


Average: -0.64111 V
Midpoint (-0.6585 V, -0.4855 V) = 113.3 K
Average: -0.67589 V
Signal
Temperature (V)


Disk 1 Run 4 Total
-0.62
-0.63
-0.64
-0.65
-0.66
-0.67
-0.68
-0.69
-0.7
-0.71
-0.72
-0.73
-0.74
-0.75
-0.76
-0.77
-0.78
-0.79
-0.8
-0.81
-0.82
Signal (V)
Time (s)


Disk 1 Run 4
Signal (V)
Temperature (V)
Average: -0.6663 V
Midpoint (-0.709 V, -0.4905 V) = 111 K
Average: -0.75175 V


Disk 1 Run 5 Total
Signal (V)
-0.13
-0.14
-0.15
-0.16
-0.17
-0.18
-0.19
-0.2
-0.21
-0.22
-0.23
-0.24
-0.25
-0.26
-0.27
-0.28
-0.29
-0.3
-0.31
-0.32
-0.33
-0.34
-0.35
-0.36
-0.37
-0.38
-0.39
-0.4
Time (s)
0
4
8
12
16
20
24
28
32
36
40
44
48
52
56
60
64
68
72
76
80
84
88
92
96
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176
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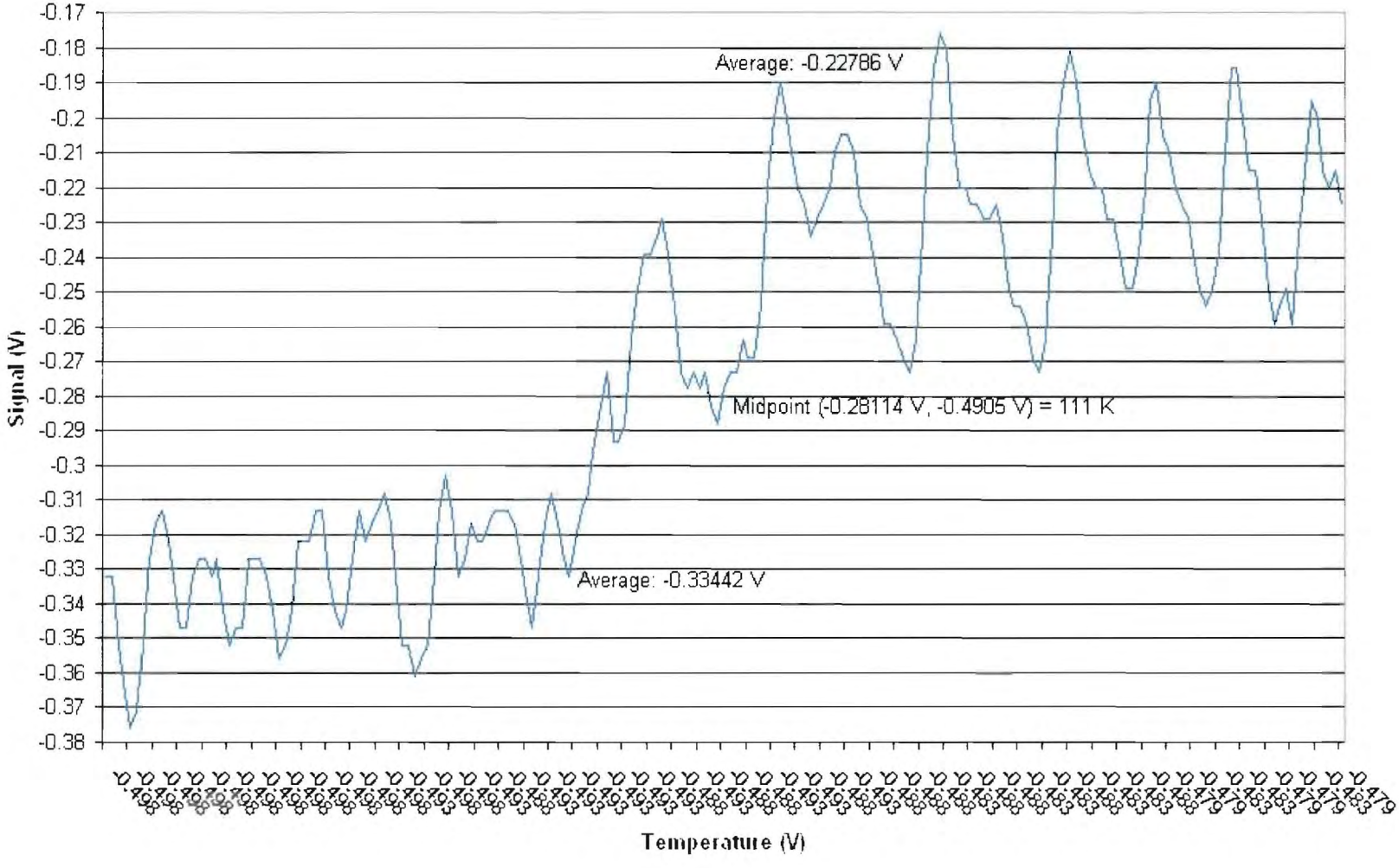
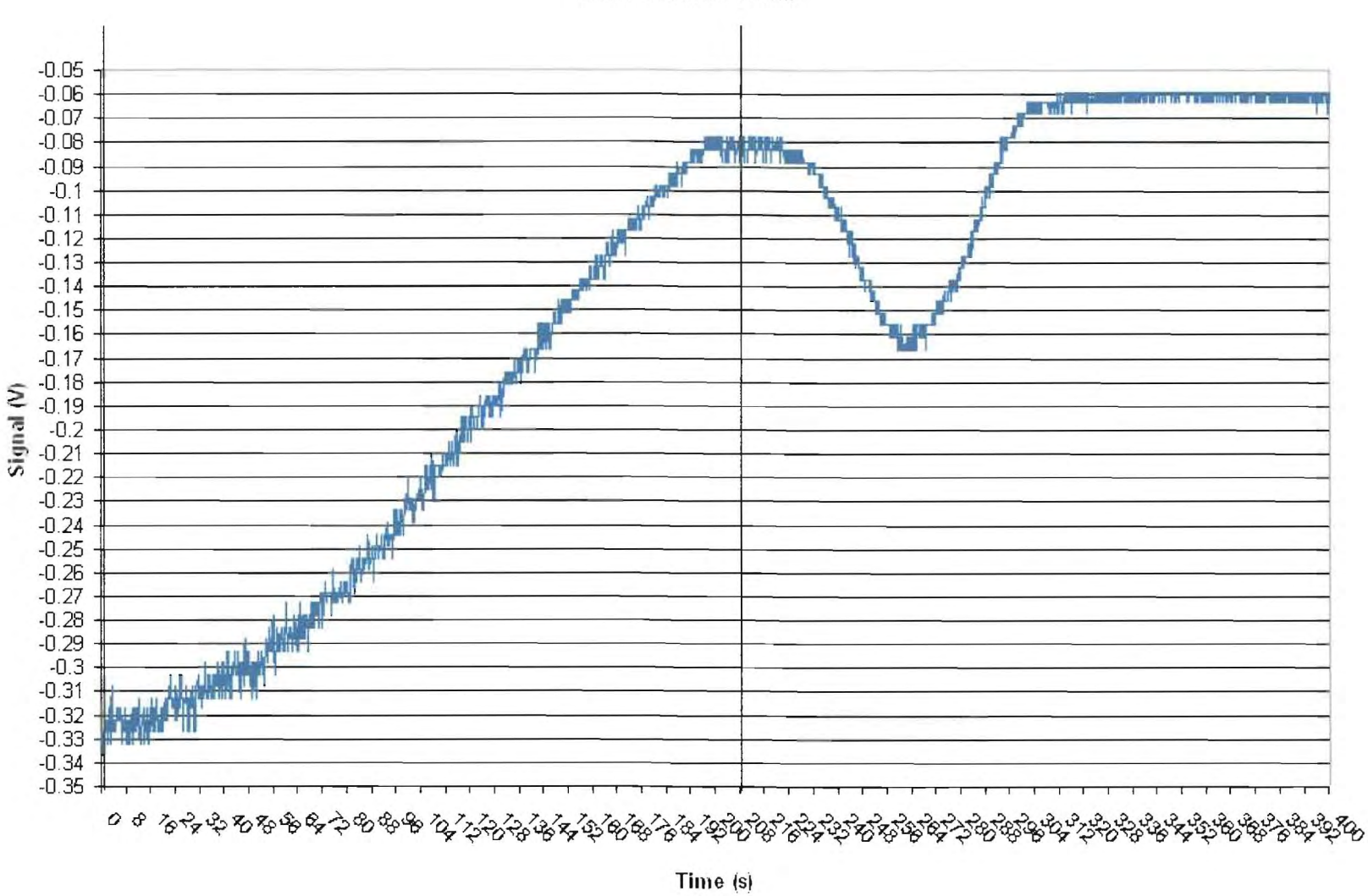
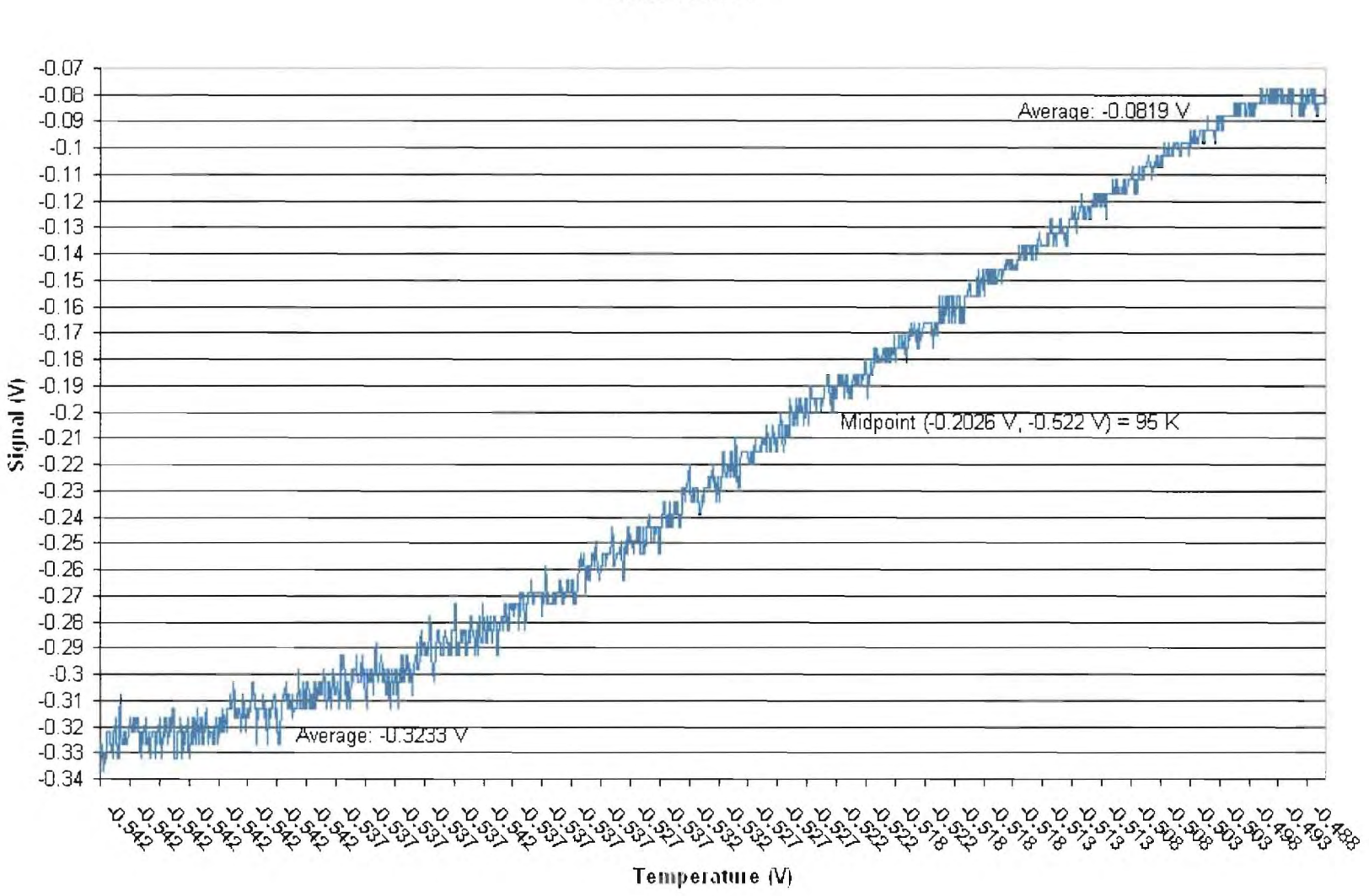
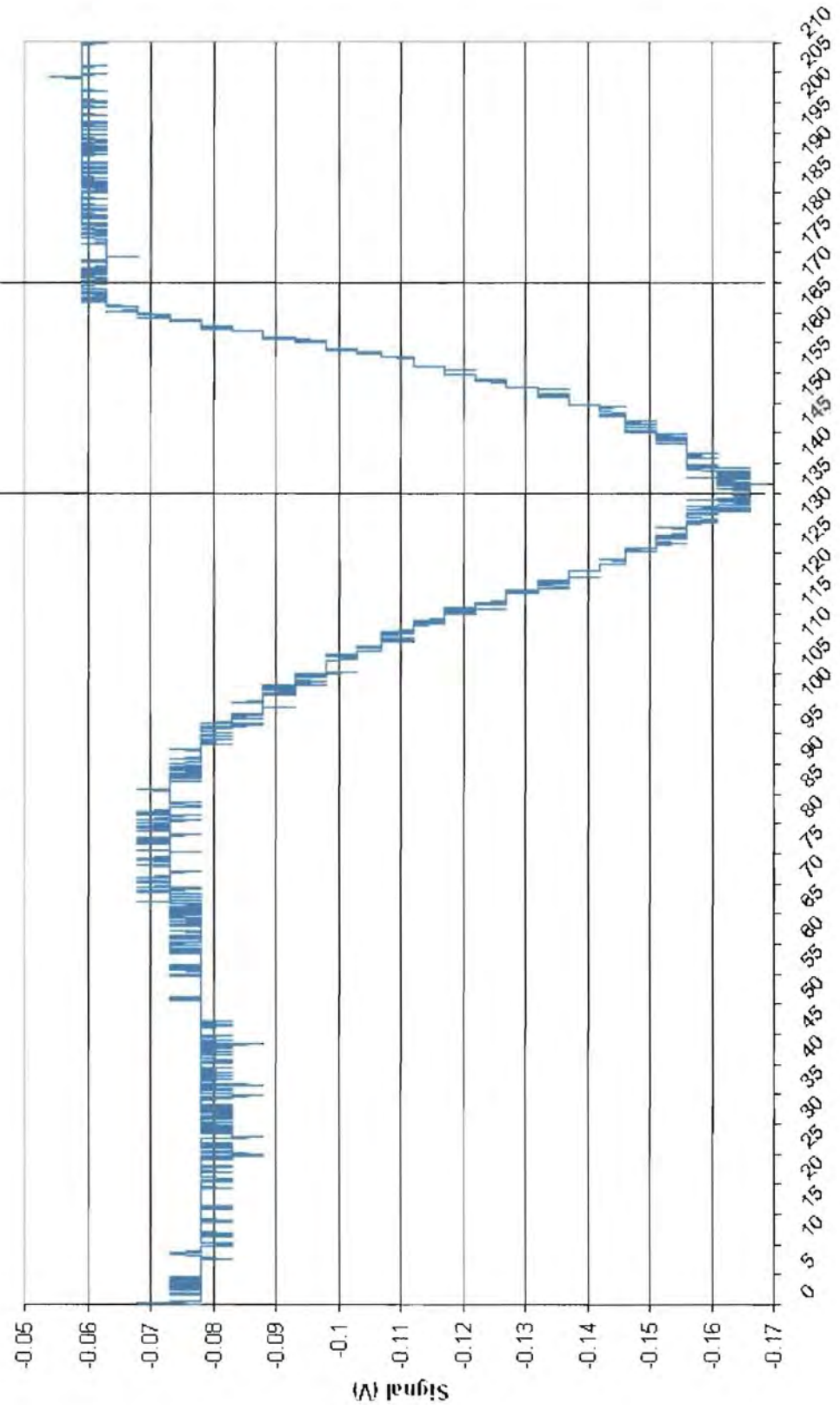
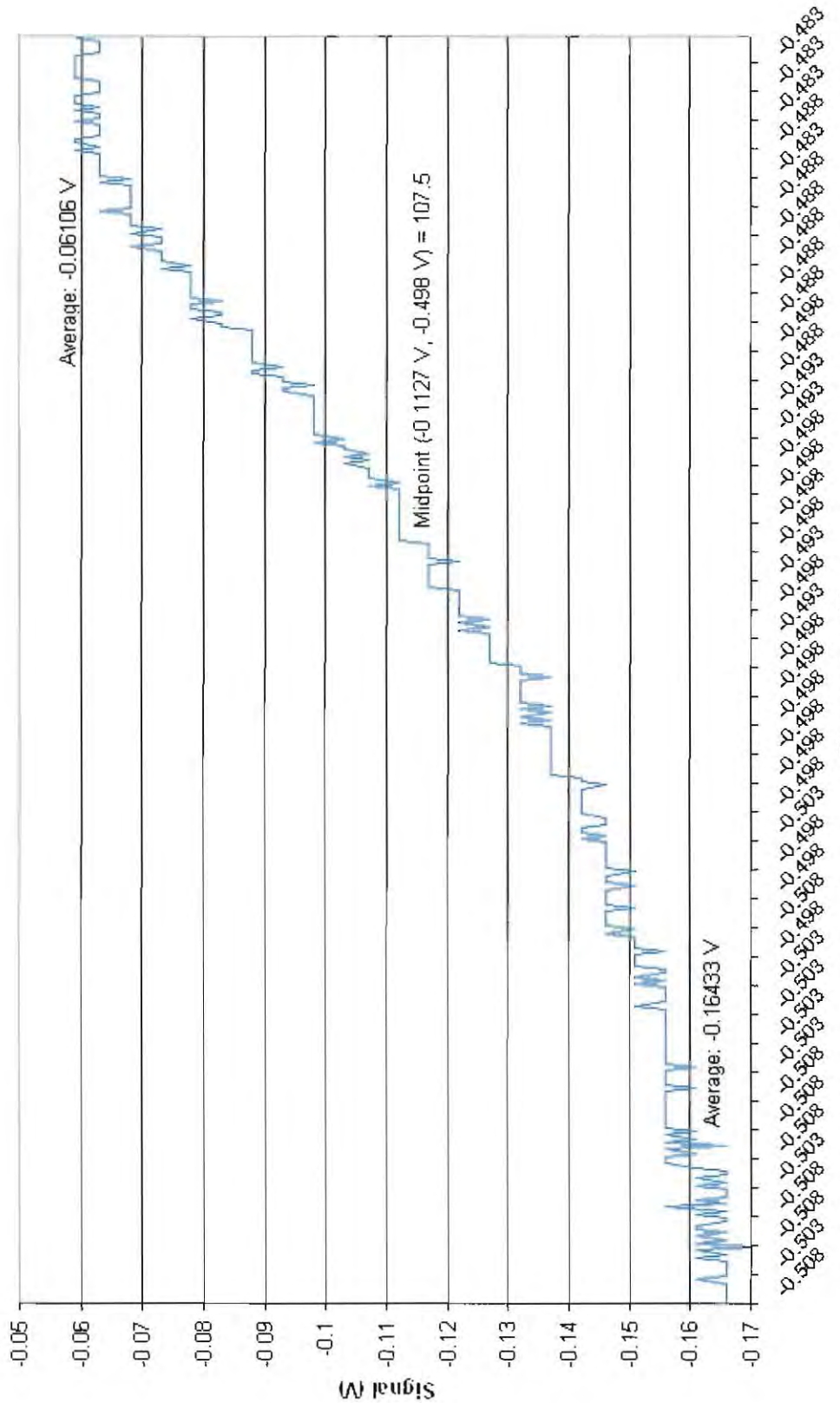
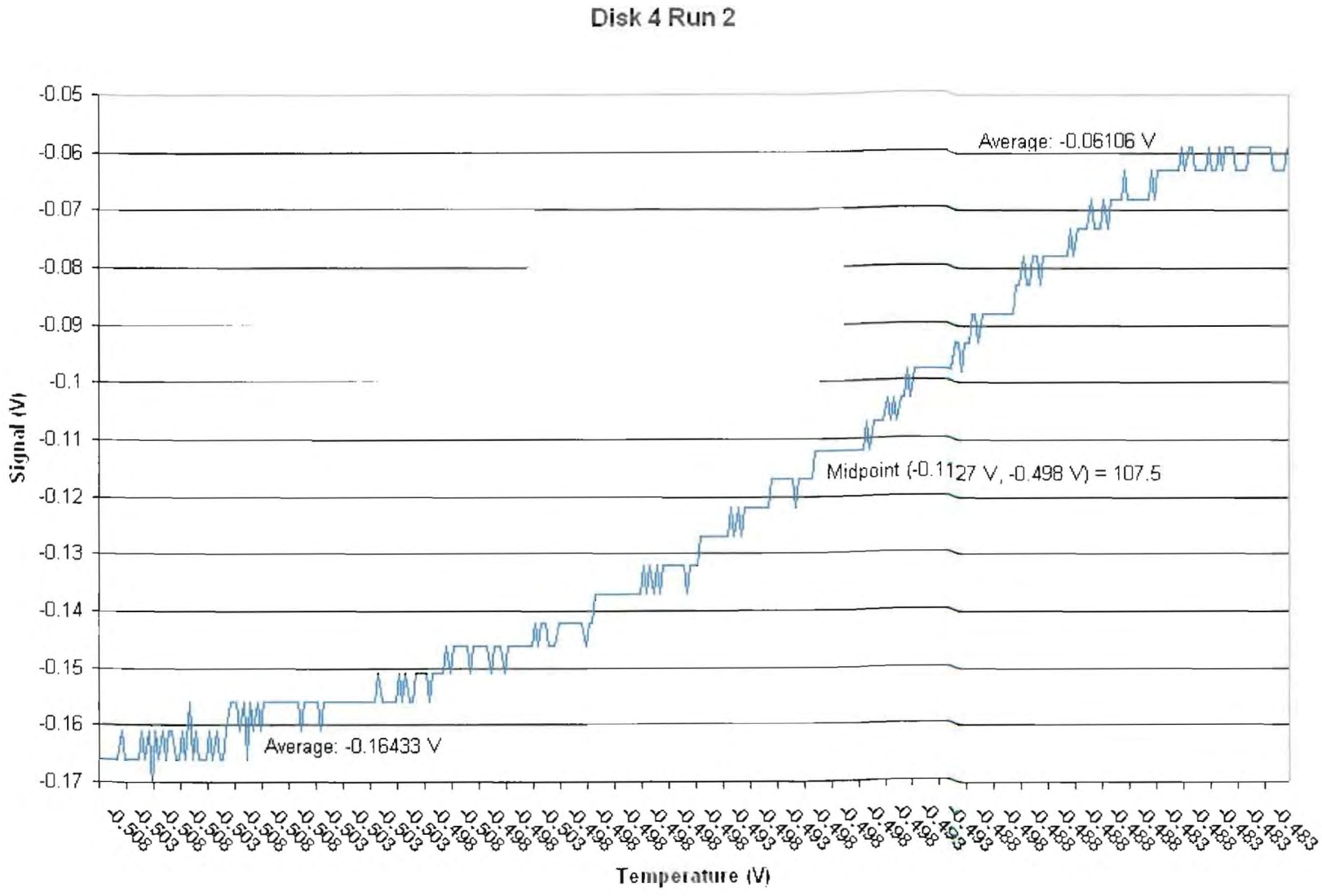

Disk 1 Run 5
Average: -0.22786 V
Midpoint (-0.28114 V, -0.4905 V) = 111 K
Average: -0.33442 V
Signal (V)
Temperature (V)
-0.17
-0.18
-0.19
-0.2
-0.21
-0.22
-0.23
-0.24
-0.25
-0.26
-0.27
-0.28
-0.29
-0.3
-0.31
-0.32
-0.33
-0.34
-0.35
-0.36
-0.37
-0.38

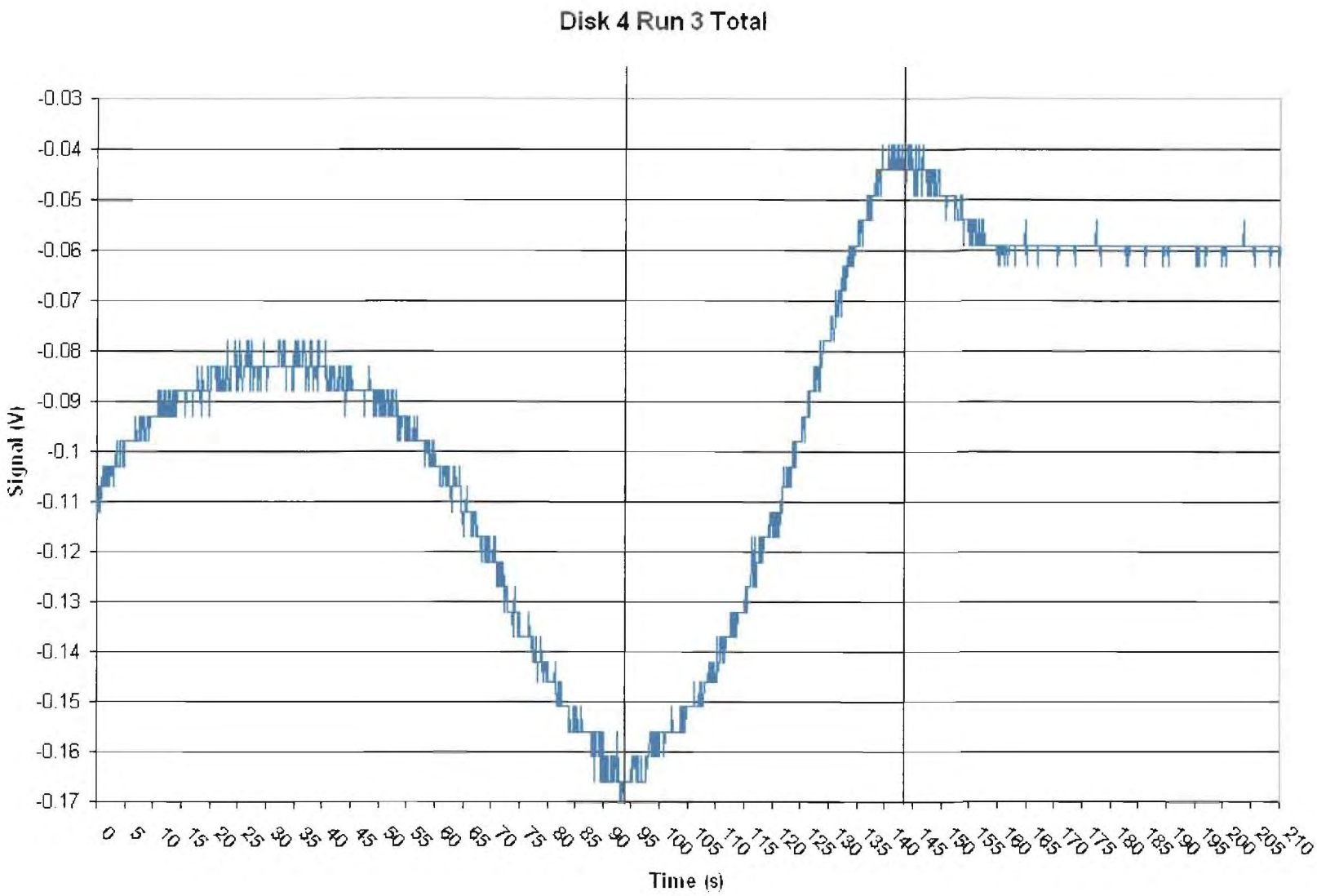

Disk 4 Run 1 Total
Signal (V)
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
-0.11
-0.12
-0.13
-0.14
-0.15
-0.16
-0.17
-0.18
-0.19
-0.2
-0.21
-0.22
-0.23
-0.24
-0.25
-0.26
-0.27
-0.28
-0.29
-0.3
-0.31
-0.32
-0.33
-0.34
-0.35
0 8 16 24 32 40 48 56 64 72 80 88 96 104 112 120 128 136 144 152 160 168 176 184 192 200 208 216 224 232 240 248 256 264 272 280 288 296 304 312 320 328 336 344 352 360 368 376 384 392 400
Time (s)

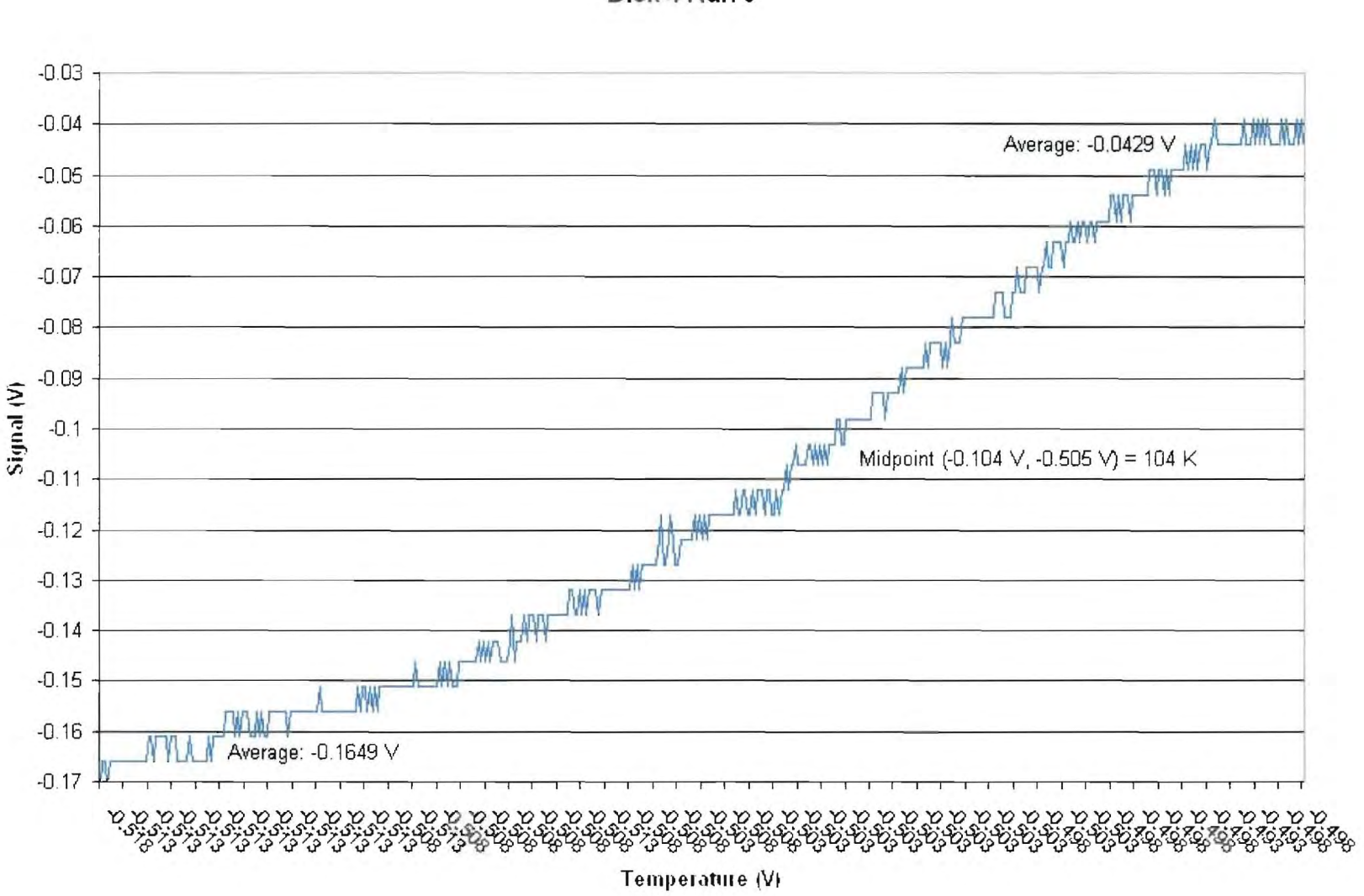

Disk 4 Run 1
Average: -0.0819 V
Midpoint (-0.2026 V, -0.522 V) = 95 K
Average: -0.3233 V
Signal (V)
Temperature (V)

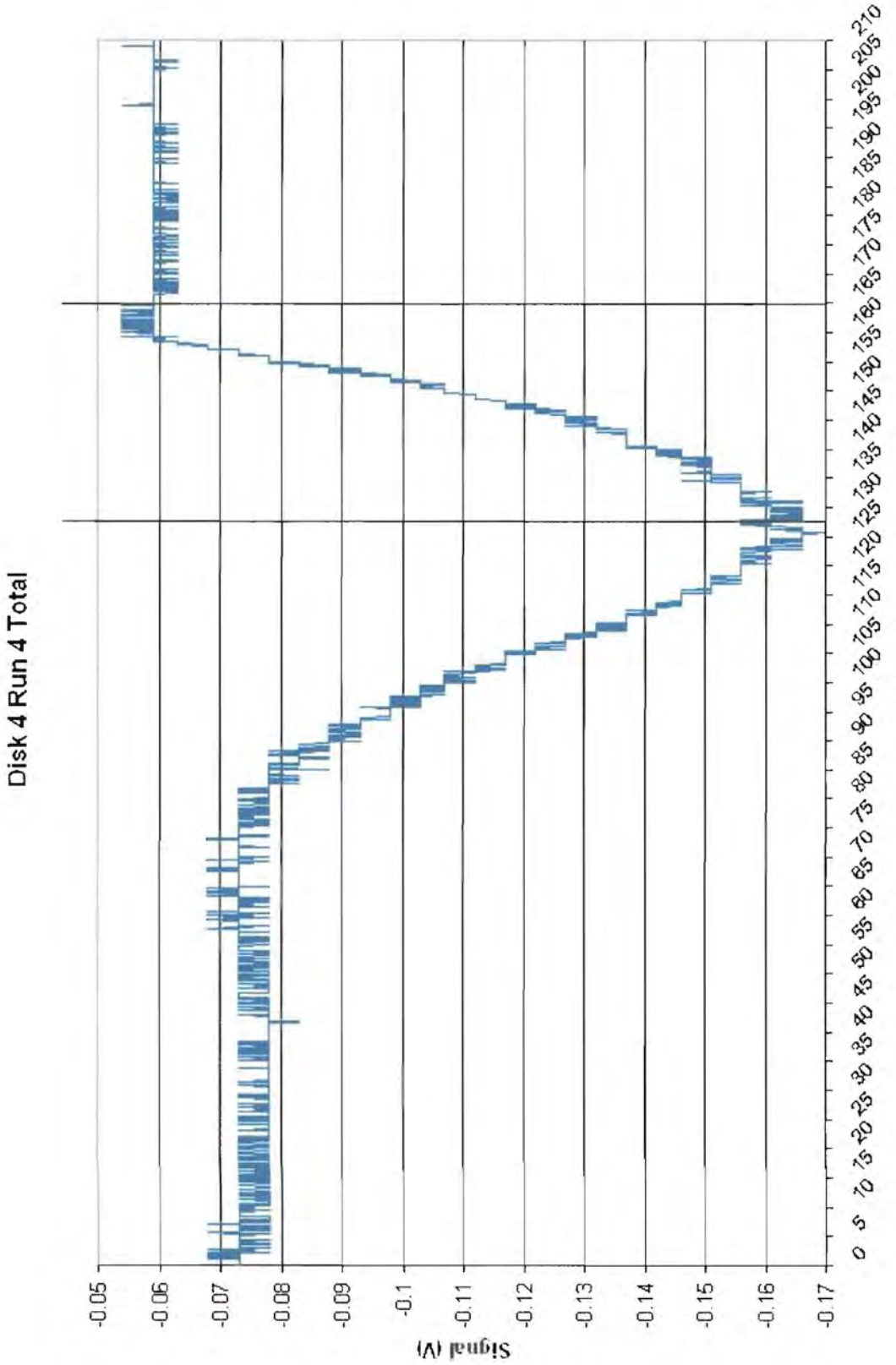

Disk 4 Run 2 Total
Signal (V)
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
-0.11
-0.12
-0.13
-0.14
-0.15
-0.16
-0.17
Time (s)
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
105
110
115
120
125
130
135
140
145
150
155
160
165
170
175
180
185
190
195
200
205
210

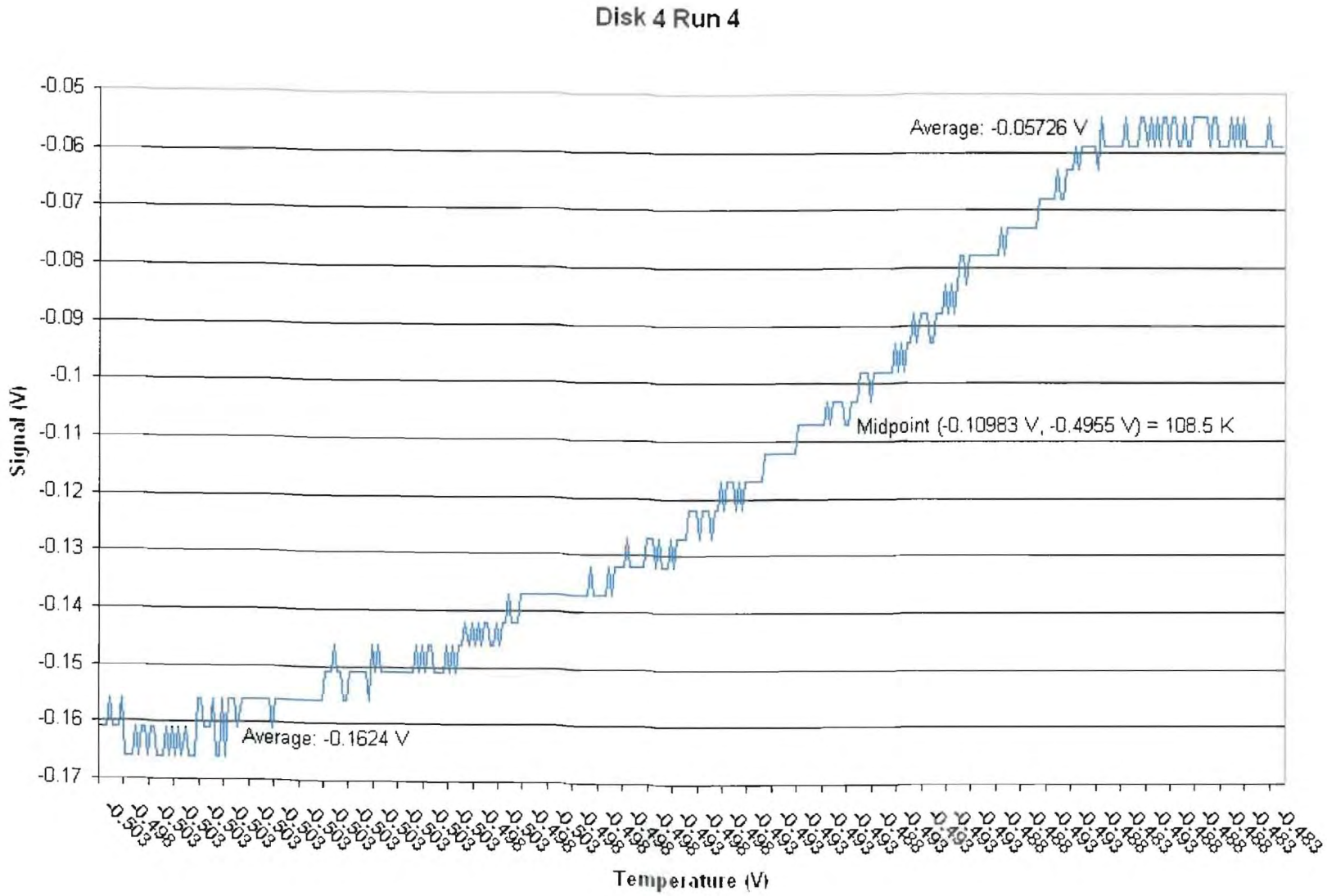

Disk 4 Run 2
Average: -0.06106 V
Midpoint (-0.1127 V, -0.498 V) = 107.5
Average: -0.16433 V
Signal (V)
Temperature (V)
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
-0.11
-0.12
-0.13
-0.14
-0.15
-0.16
-0.17
0.483
0.488
0.493
0.498
0.503
0.508

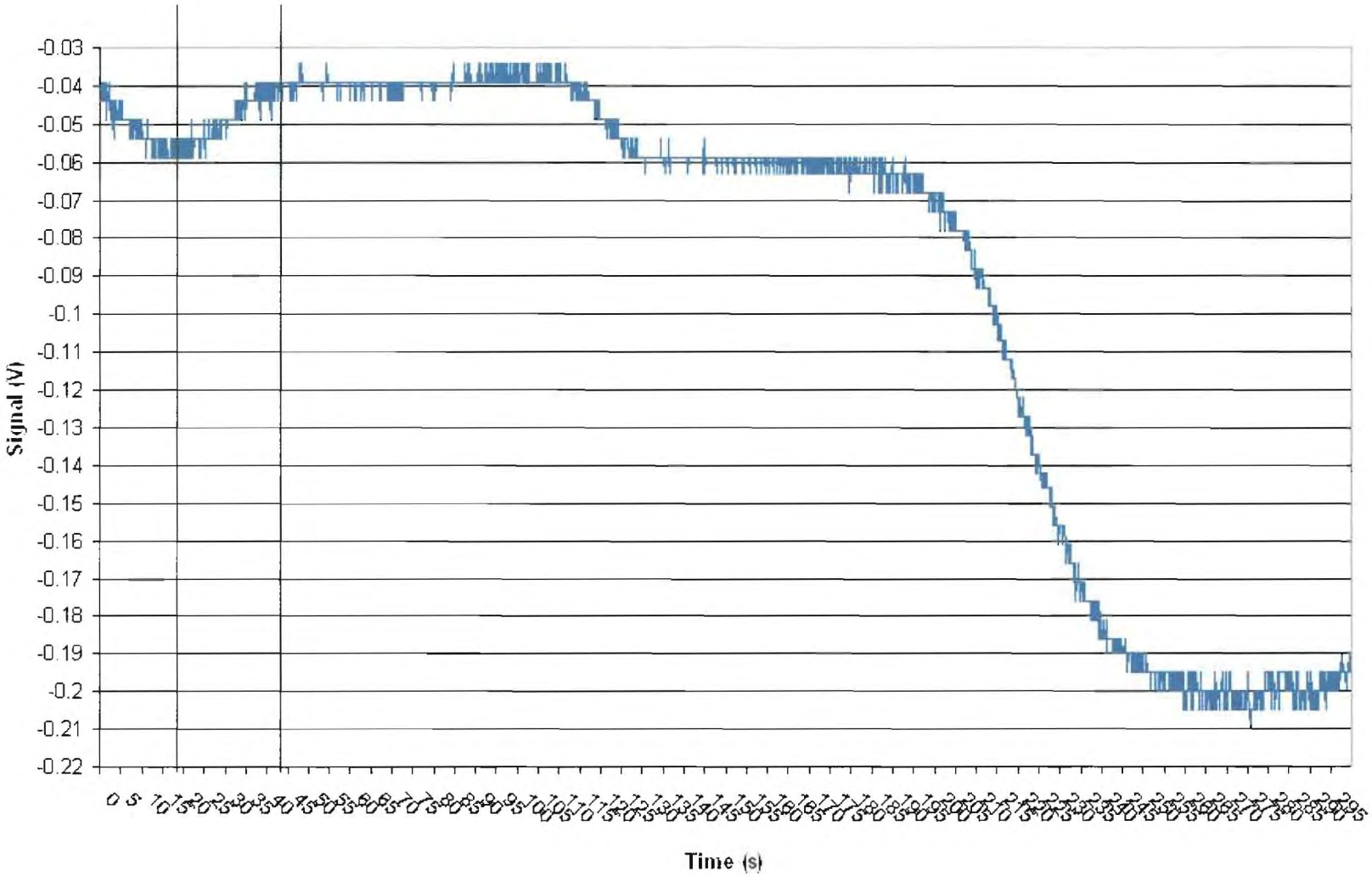

Disk 4 Run 2
Signal (V)
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
-0.11
-0.12
-0.13
-0.14
-0.15
-0.16
-0.17
Average: -0.06106 V
Midpoint (-0.1127 V, -0.498 V) = 107.5
Average: -0.16433 V
0.508
0.503
0.498
0.493
0.488
0.483
Temperature (V)


Disk 4 Run 3 Total
Signal (V)
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
-0.11
-0.12
-0.13
-0.14
-0.15
-0.16
-0.17
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140 145 150 155 160 165 170 175 180 185 190 195 200 205 210
Time (s)

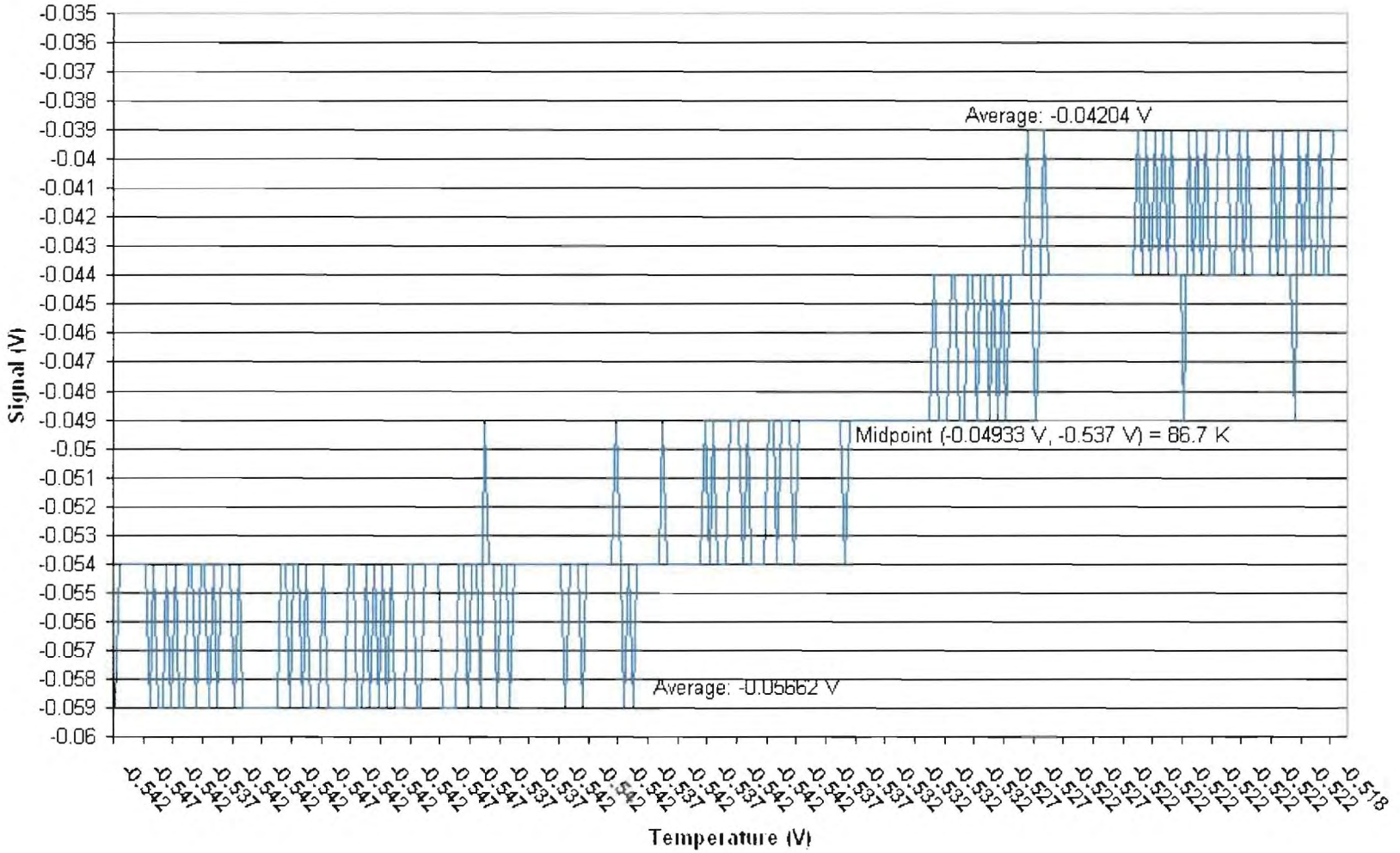

Disk 4 Run 3
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
-0.11
-0.12
-0.13
-0.14
-0.15
-0.16
-0.17
Signal (V)
Average: -0.0429 V
Midpoint (-0.104 V, -0.505 V) = 104 K
Average: -0.1649 V
Temperature (V)

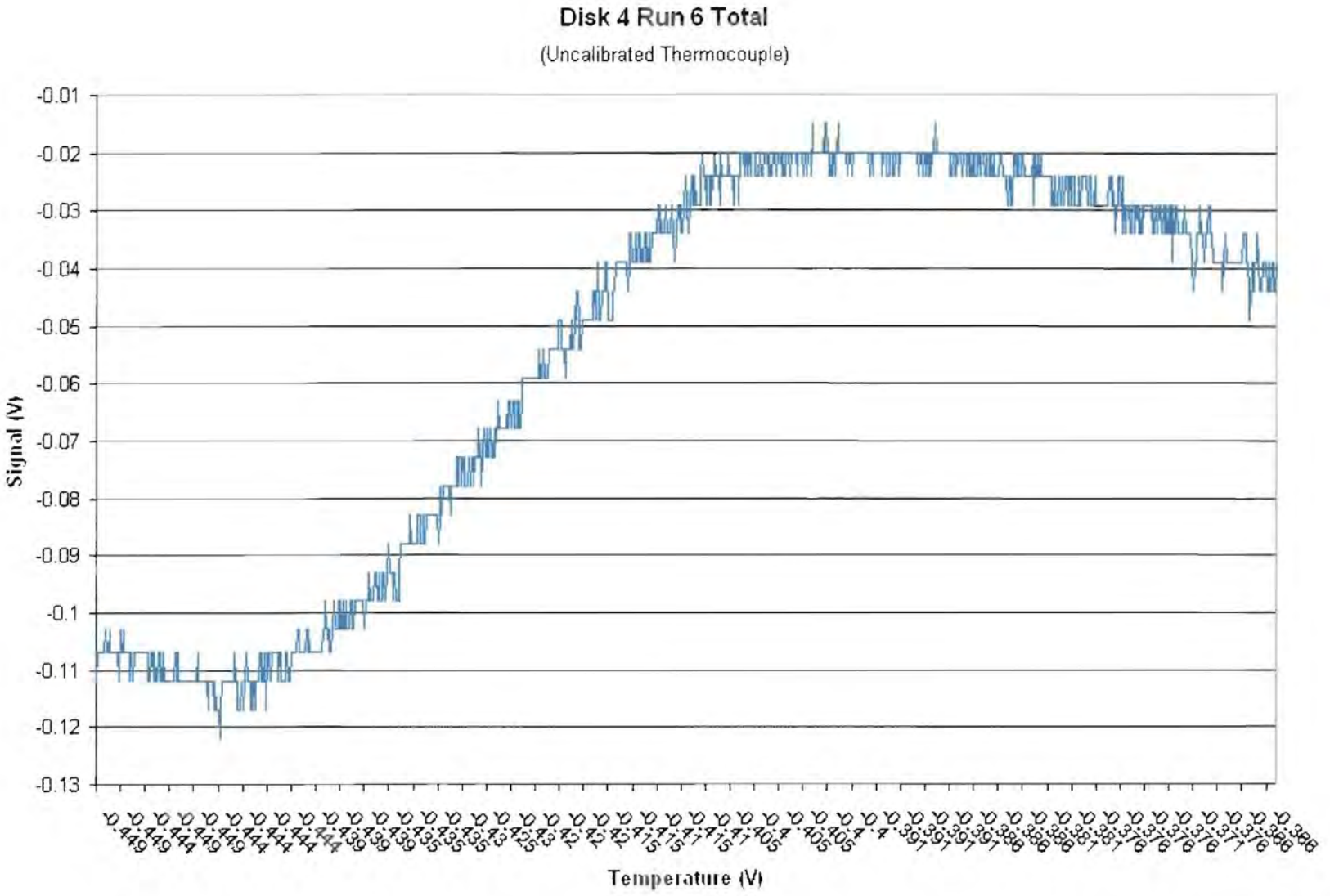

Disk 4 Run 4 Total
Signal (V)
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
-0.11
-0.12
-0.13
-0.14
-0.15
-0.16
-0.17
Time (s)
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
105
110
115
120
125
130
135
140
145
150
155
160
165
170
175
180
185
190
195
200
205
210

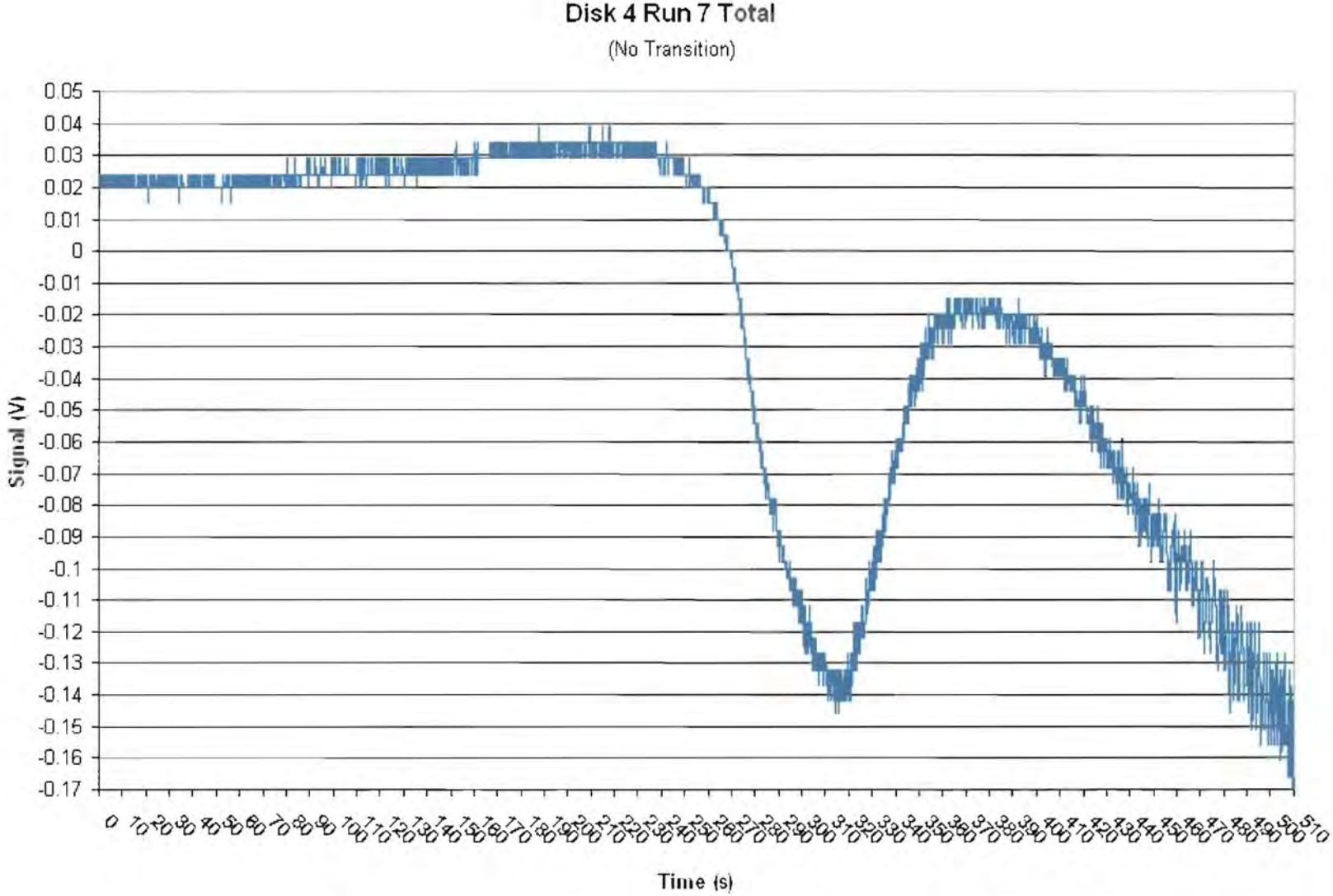

Disk 4 Run 4
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
-0.11
-0.12
-0.13
-0.14
-0.15
-0.16
-0.17
Signal (V)
Average: -0.05726 V
Midpoint (-0.10983 V, -0.4955 V) = 108.5 K
Average: -0.1624 V
Temperature (V)

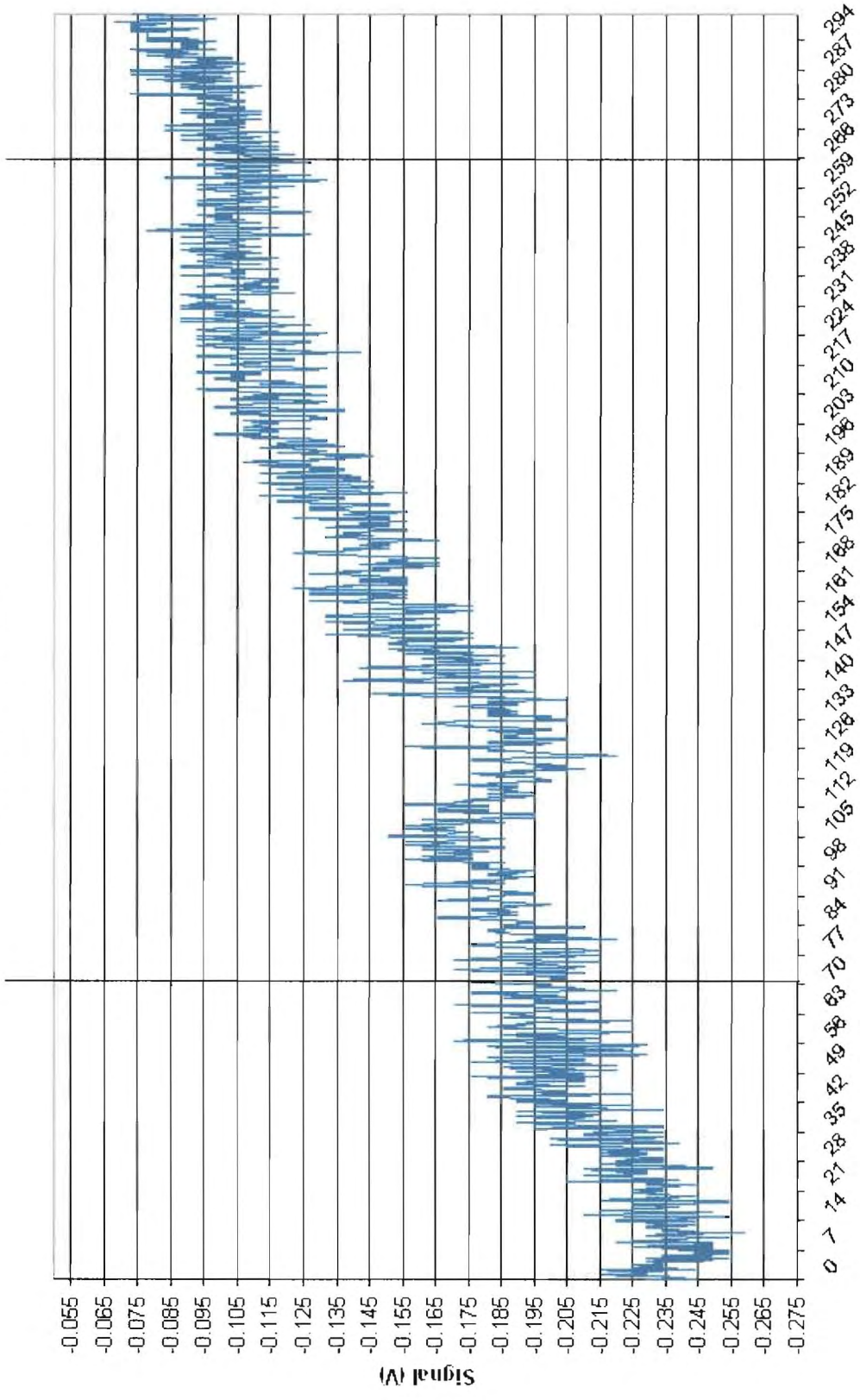

Disk 4 Run 5 Total
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
-0.11
-0.12
-0.13
-0.14
-0.15
-0.16
-0.17
-0.18
-0.19
-0.2
-0.21
-0.22
Signal (V)
Time (s)

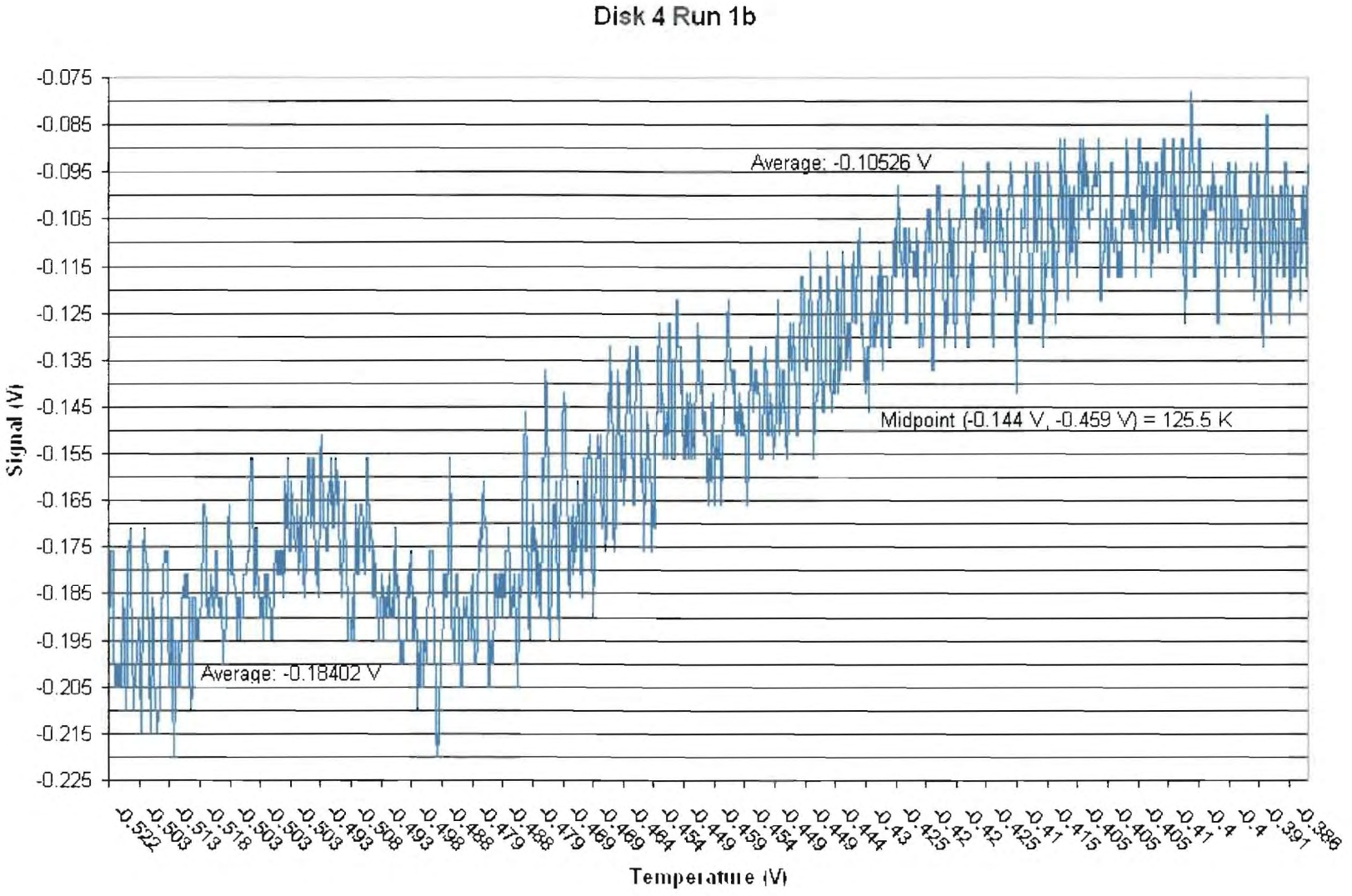
Disk 4 Run 5

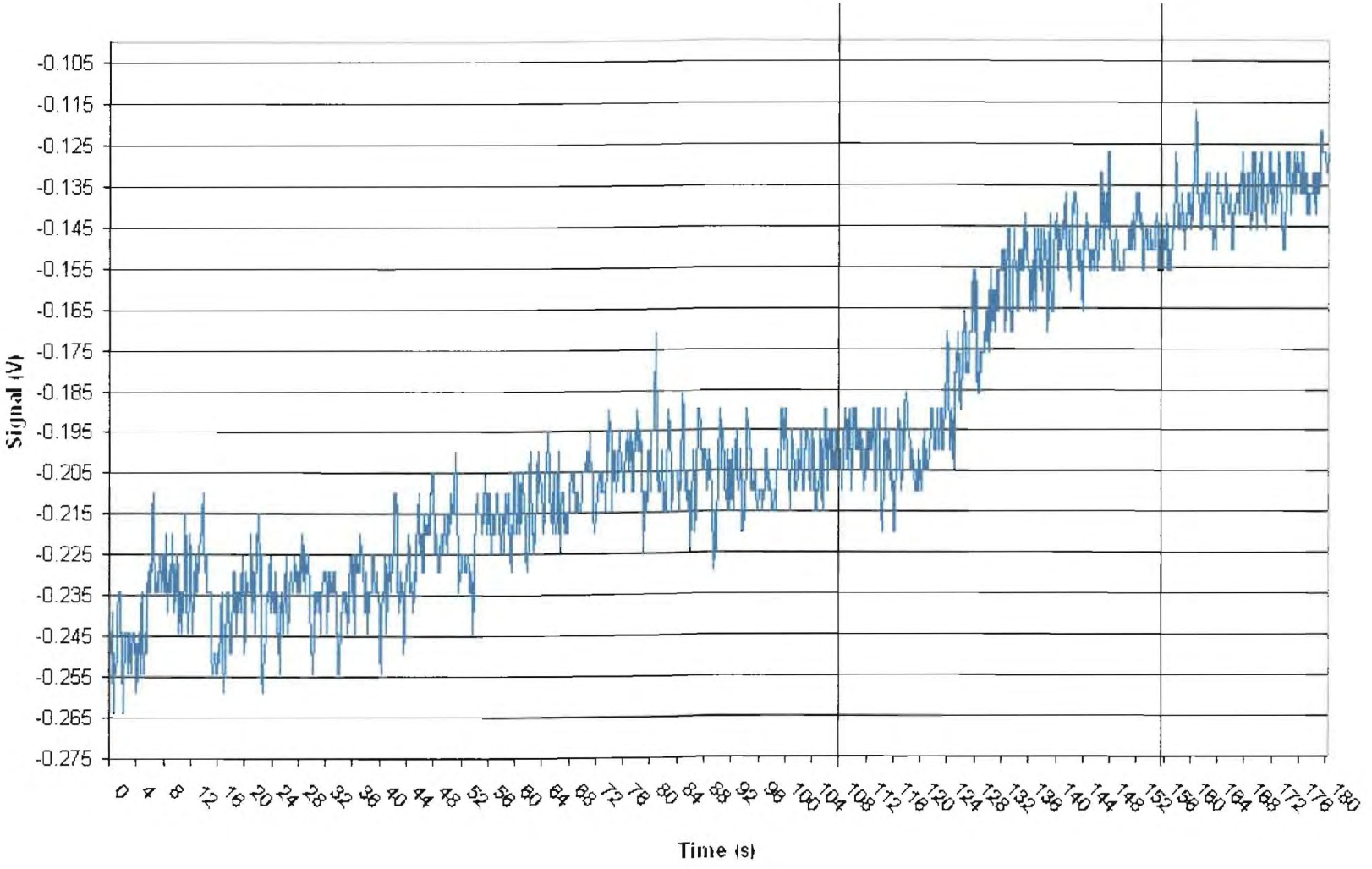

Average: -0.04204 V
Midpoint (-0.04933 V, -0.537 V) = 86.7 K
Average: -0.05662 V
Signal (V)
Temperature (V)

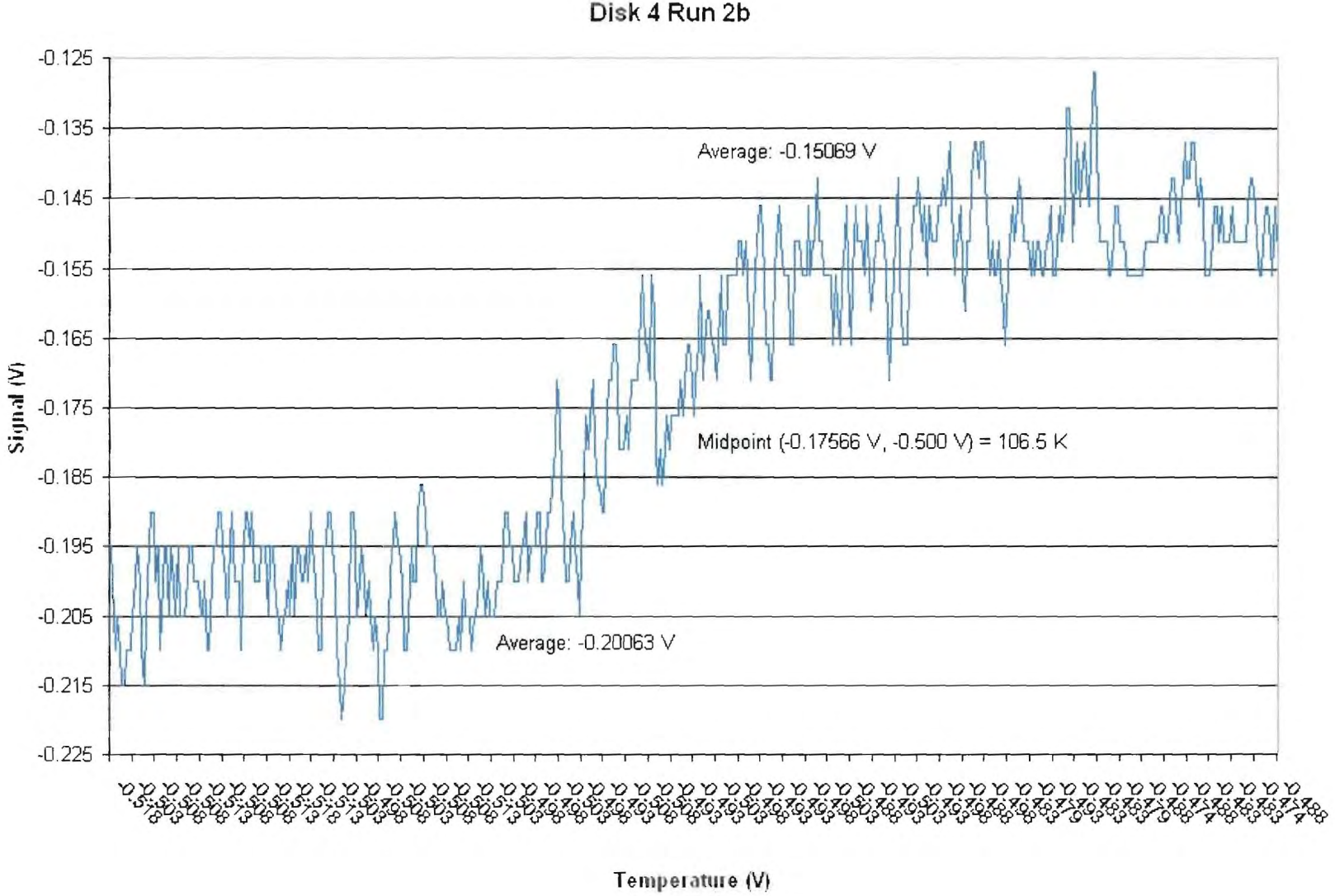

Disk 4 Run 6 Total
(Uncalibrated Thermocouple)
Signal (V)
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
-0.11
-0.12
-0.13
Temperature (V)

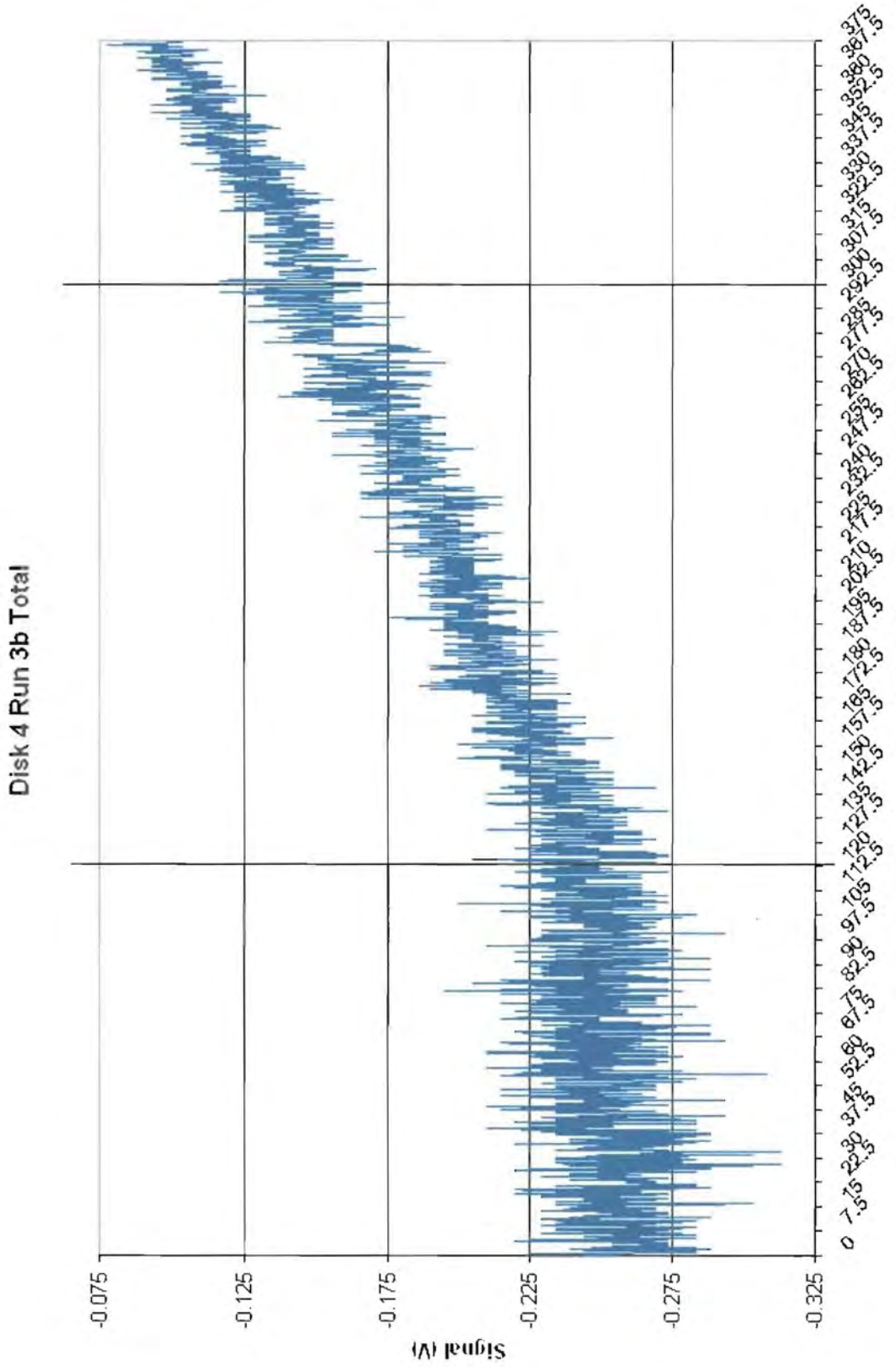

Disk 4 Run 7 Total
(No Transition)
Signal (V)
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
-0.11
-0.12
-0.13
-0.14
-0.15
-0.16
-0.17
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220 230 240 250 260 270 280 290 300 310 320 330 340 350 360 370 380 390 400 410 420 430 440 450 460 470 480 490 500 510
Time (s)

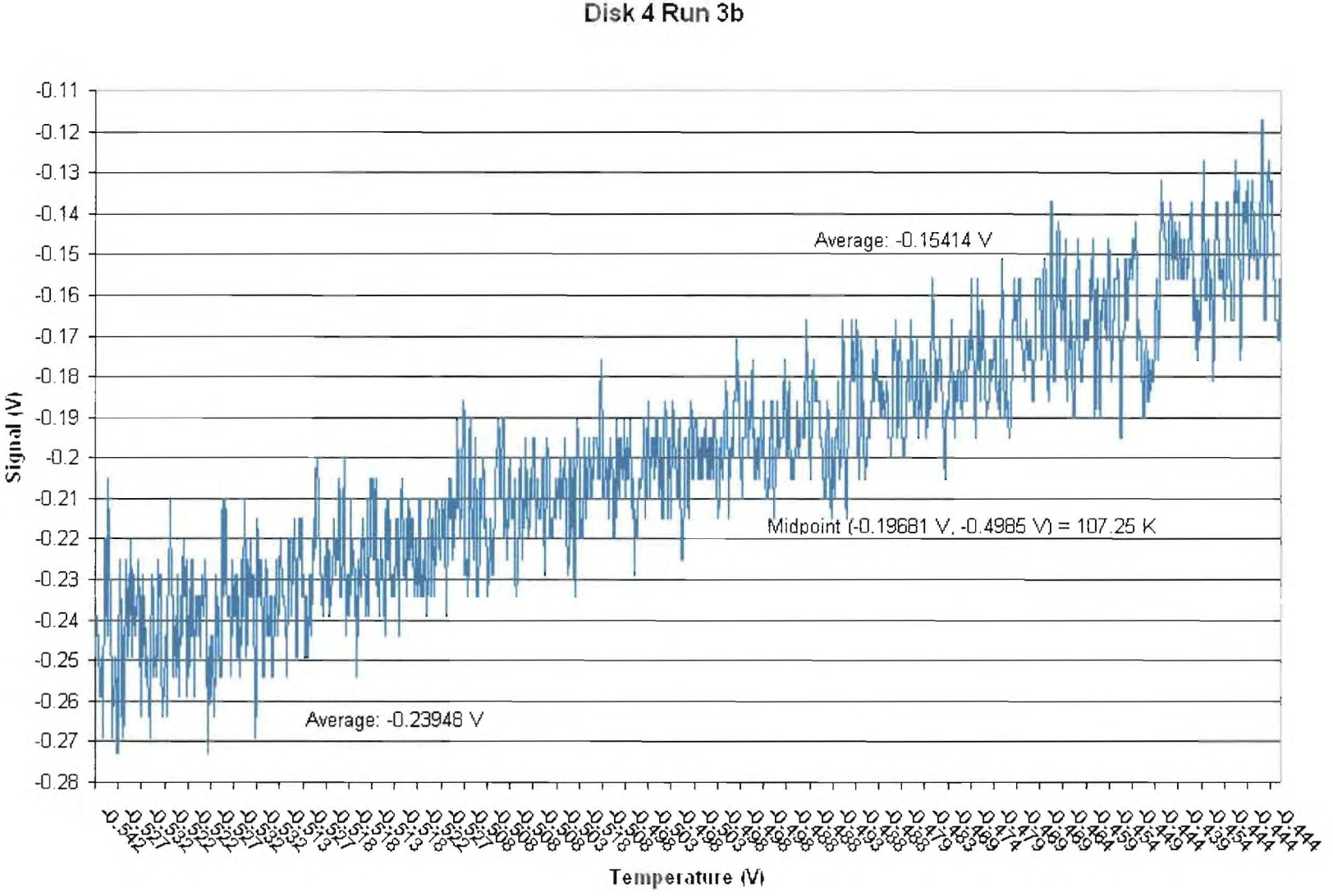

Disk 4 Run 1b Total
Signal (V)
-0.055
-0.065
-0.075
-0.085
-0.095
-0.105
-0.115
-0.125
-0.135
-0.145
-0.155
-0.165
-0.175
-0.185
-0.195
-0.205
-0.215
-0.225
-0.235
-0.245
-0.255
-0.265
-0.275
Time (s)
0
7
14
21
28
35
42
49
56
63
70
77
84
91
98
105
112
119
126
133
140
147
154
161
168
175
182
189
196
203
210
217
224
231
238
245
252
259
266
273
280
287
294

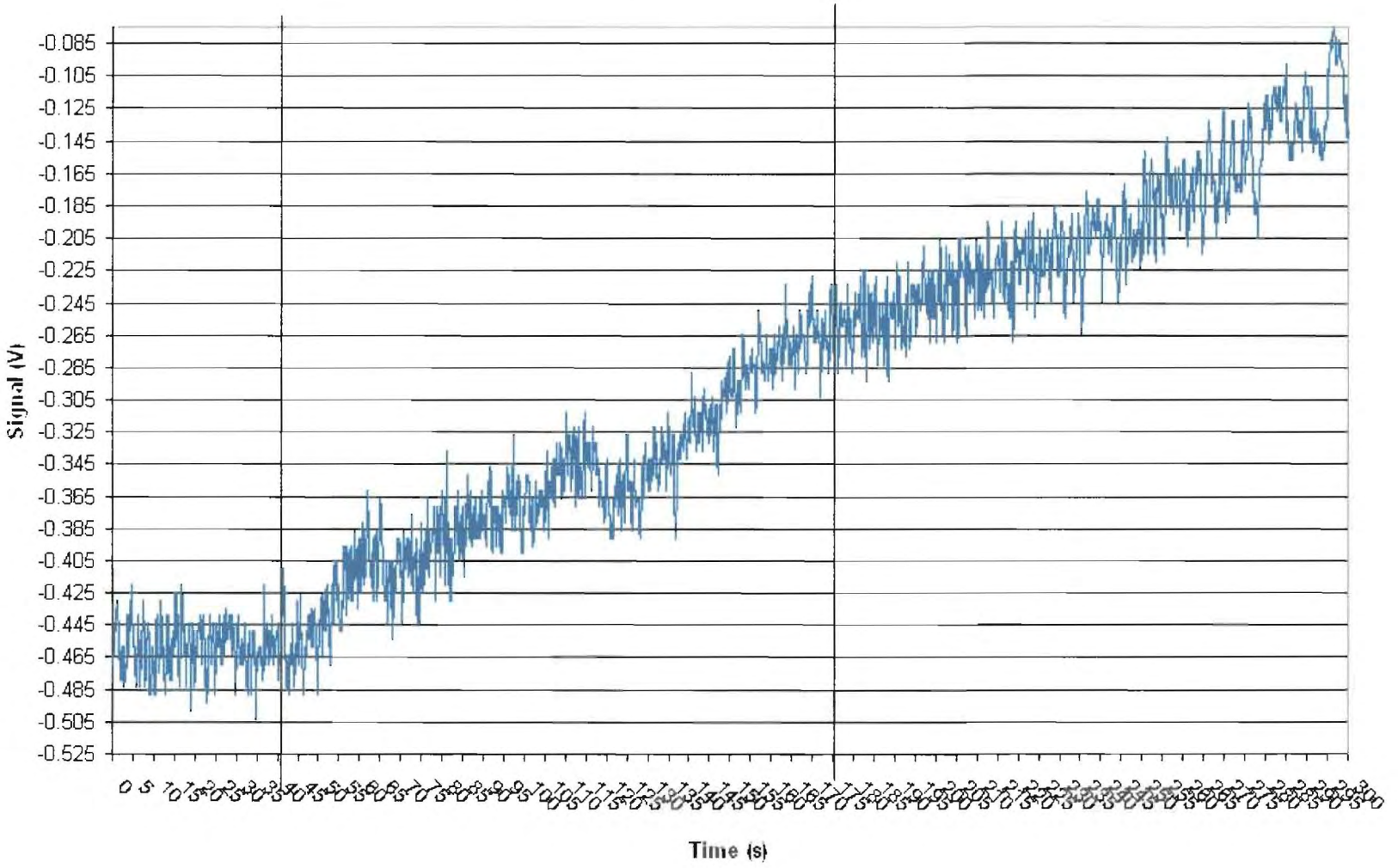

Disk 4 Run 1b
-0.075
-0.085
-0.095
-0.105
-0.115
-0.125
-0.135
-0.145
-0.155
-0.165
-0.175
-0.185
-0.195
-0.205
-0.215
-0.225
Signal (V)
Average: -0.10526 V
Midpoint (-0.144 V, -0.459 V) = 125.5 K
Average: -0.18402 V
0.522 0.503 0.513 0.518 0.503 0.503 0.503 0.493 0.508 0.493 0.498 0.488 0.479 0.488 0.479 0.469 0.469 0.464 0.454 0.449 0.459 0.454 0.449 0.449 0.444 0.43 0.425 0.42 0.42 0.425 0.41 0.415 0.405 0.405 0.405 0.41 0.4 0.4 0.391 0.386
Temperature (V)

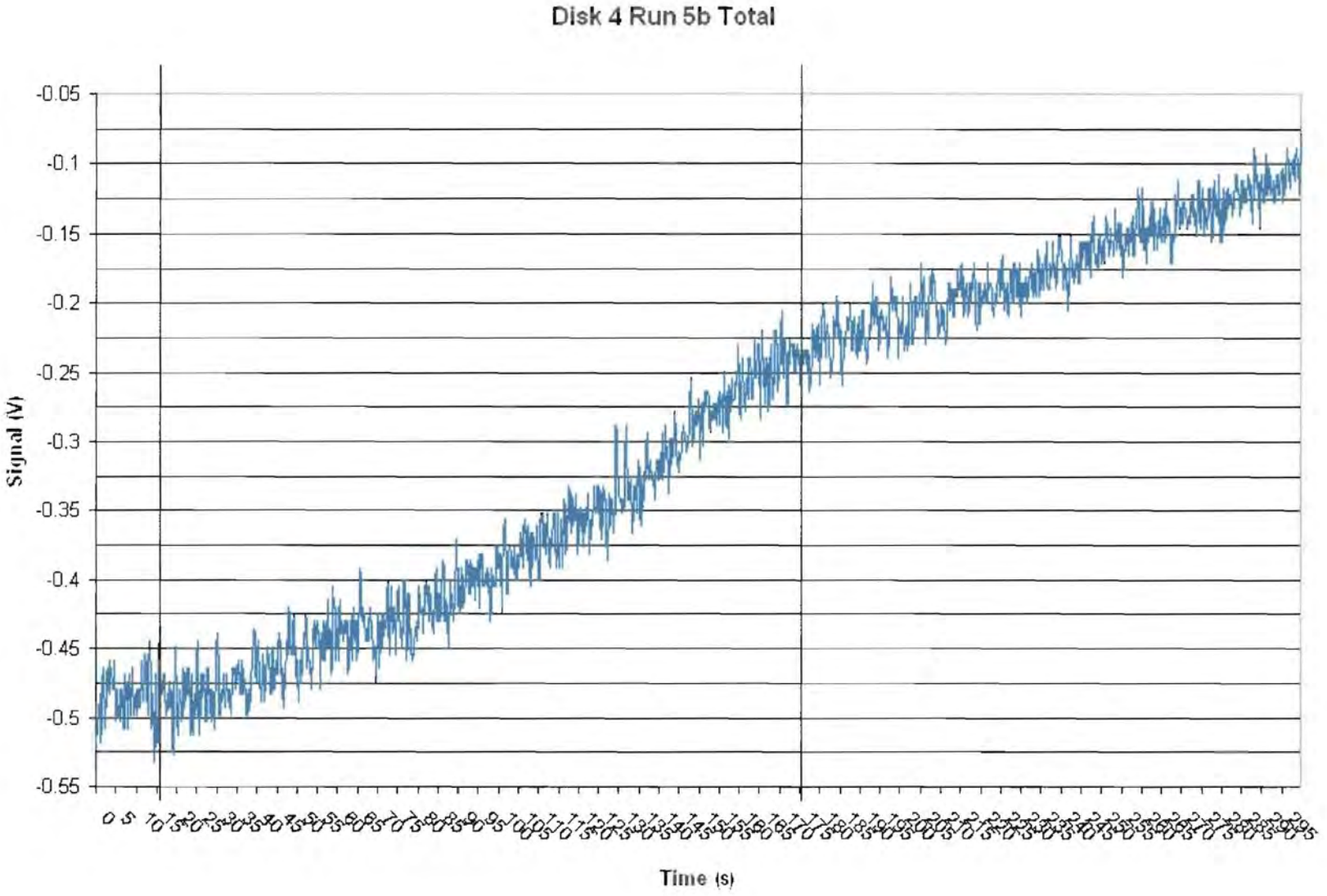

Disk 4 Run 2b Total
Signal (V)
-0.105
-0.115
-0.125
-0.135
-0.145
-0.155
-0.165
-0.175
-0.185
-0.195
-0.205
-0.215
-0.225
-0.235
-0.245
-0.255
-0.265
-0.275
0 4 8 12 16 20 24 28 32 36 40 44 48 52 56 60 64 68 72 76 80 84 88 92 96 100 104 108 112 116 120 124 128 132 136 140 144 148 152 156 160 164 168 172 176 180
Time (s)

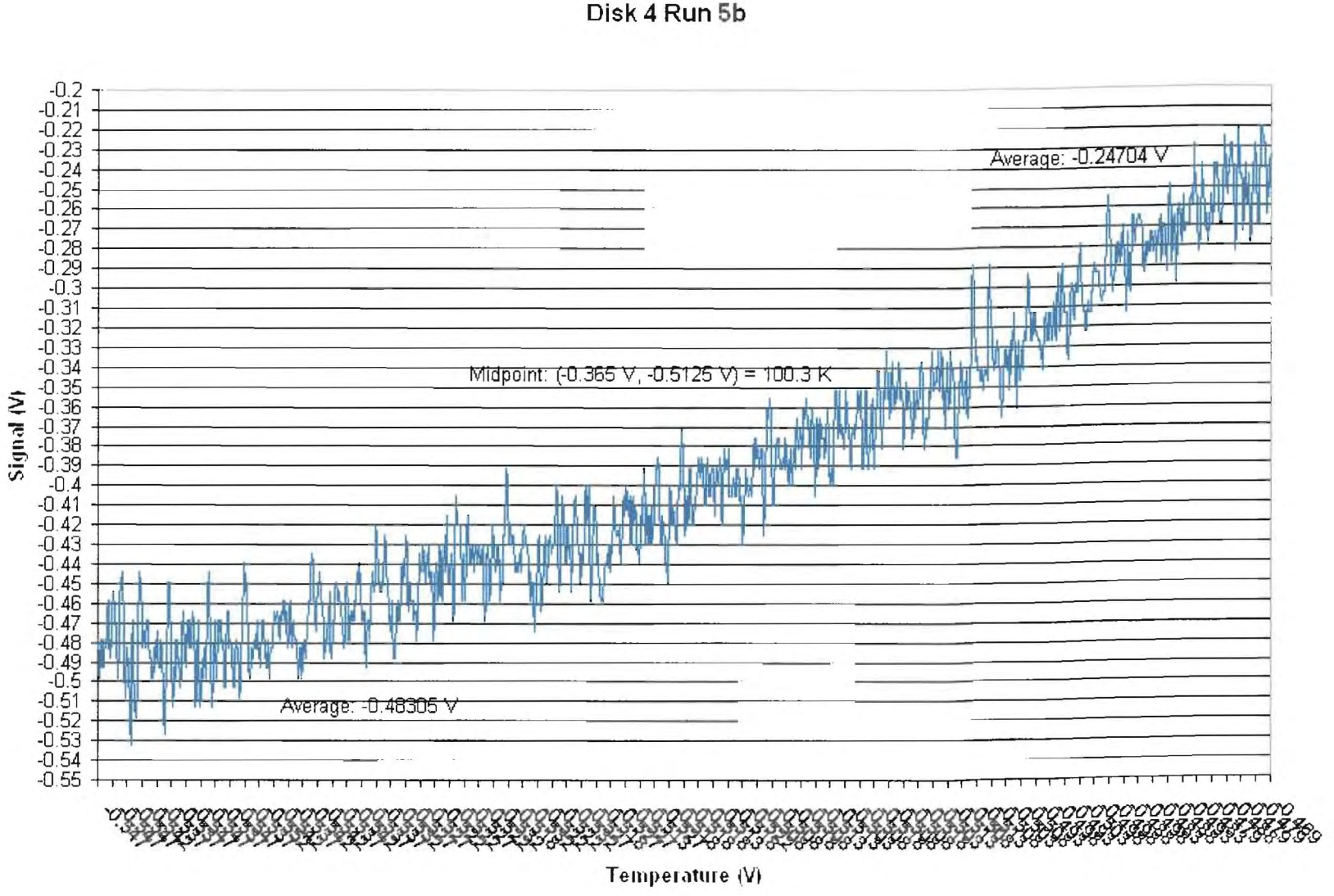

Disk 4 Run 2b
-0.125
-0.135
-0.145
-0.155
-0.165
-0.175
-0.185
-0.195
-0.205
-0.215
-0.225
Signal (V)
Average: -0.15069 V
Midpoint (-0.17566 V, -0.500 V) = 106.5 K
Average: -0.20063 V
Temperature (V)

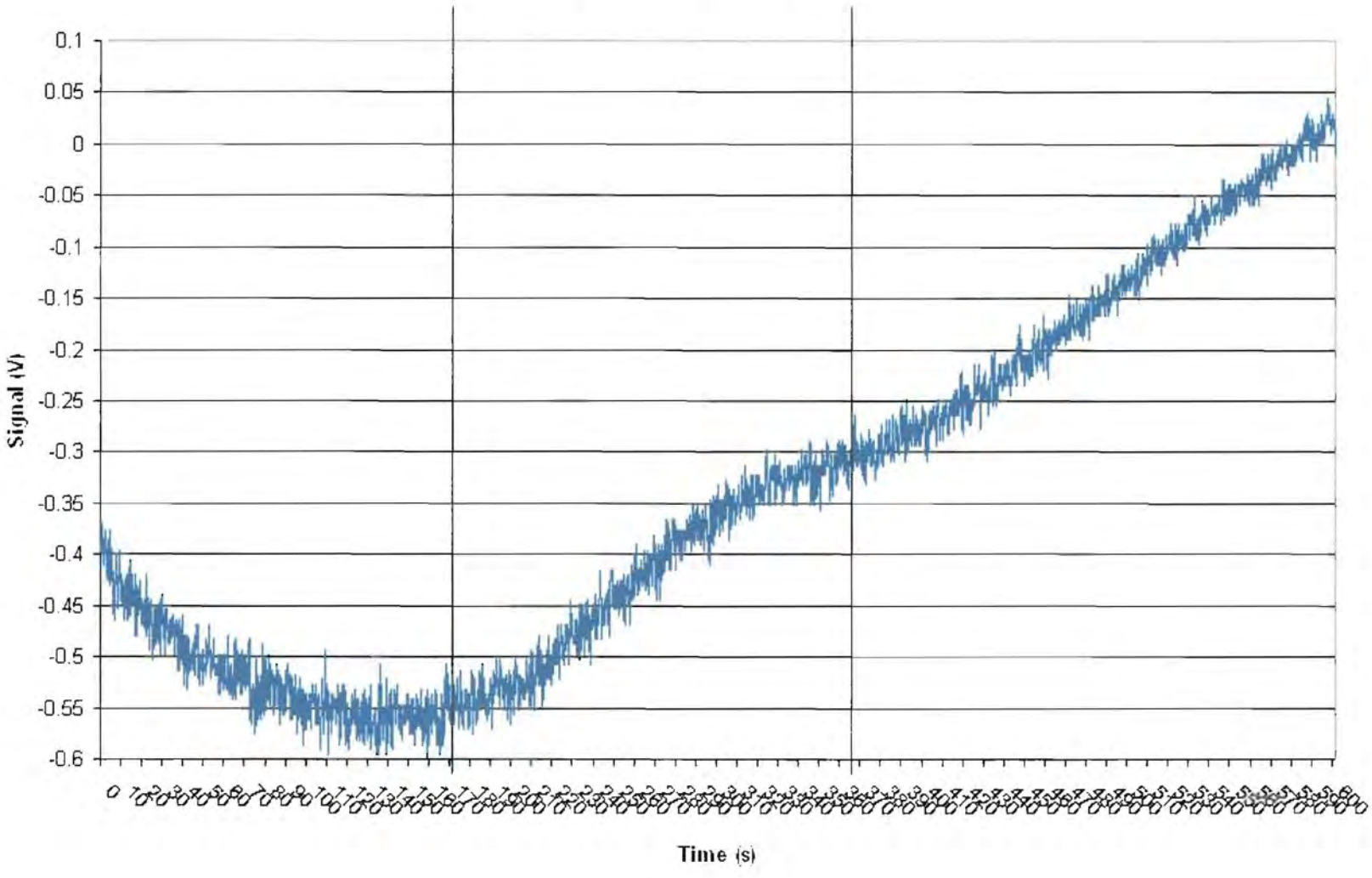

Disk 4 Run 3b Total
Signal (V)
-0.075
-0.125
-0.175
-0.225
-0.275
-0.325
Time (s)
0
7.5
15
22.5
30
37.5
45
52.5
60
67.5
75
82.5
90
97.5
105
112.5
120
127.5
135
142.5
150
157.5
165
172.5
180
187.5
195
202.5
210
217.5
225
232.5
240
247.5
255
262.5
270
277.5
285
292.5
300
307.5
315
322.5
330
337.5
345
352.5
360
367.5
375

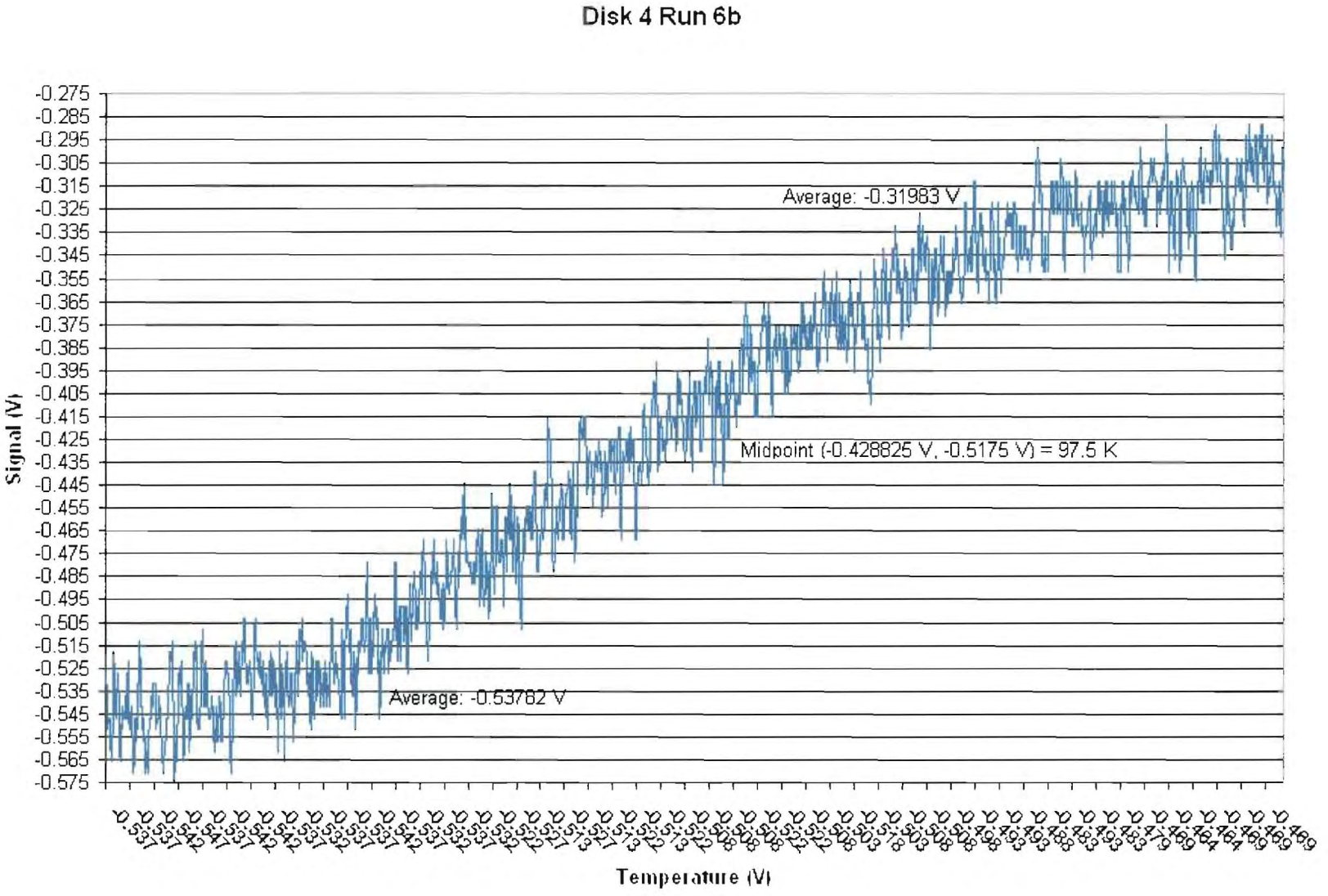

Disk 4 Run 3b
-0.11
-0.12
-0.13
-0.14
-0.15
-0.16
-0.17
-0.18
-0.19
-0.2
-0.21
-0.22
-0.23
-0.24
-0.25
-0.26
-0.27
-0.28
Signal (V)
Average: -0.15414 V
Midpoint (-0.19681 V, -0.4985 V) = 107.25 K
Average: -0.23948 V
Temperature (V)

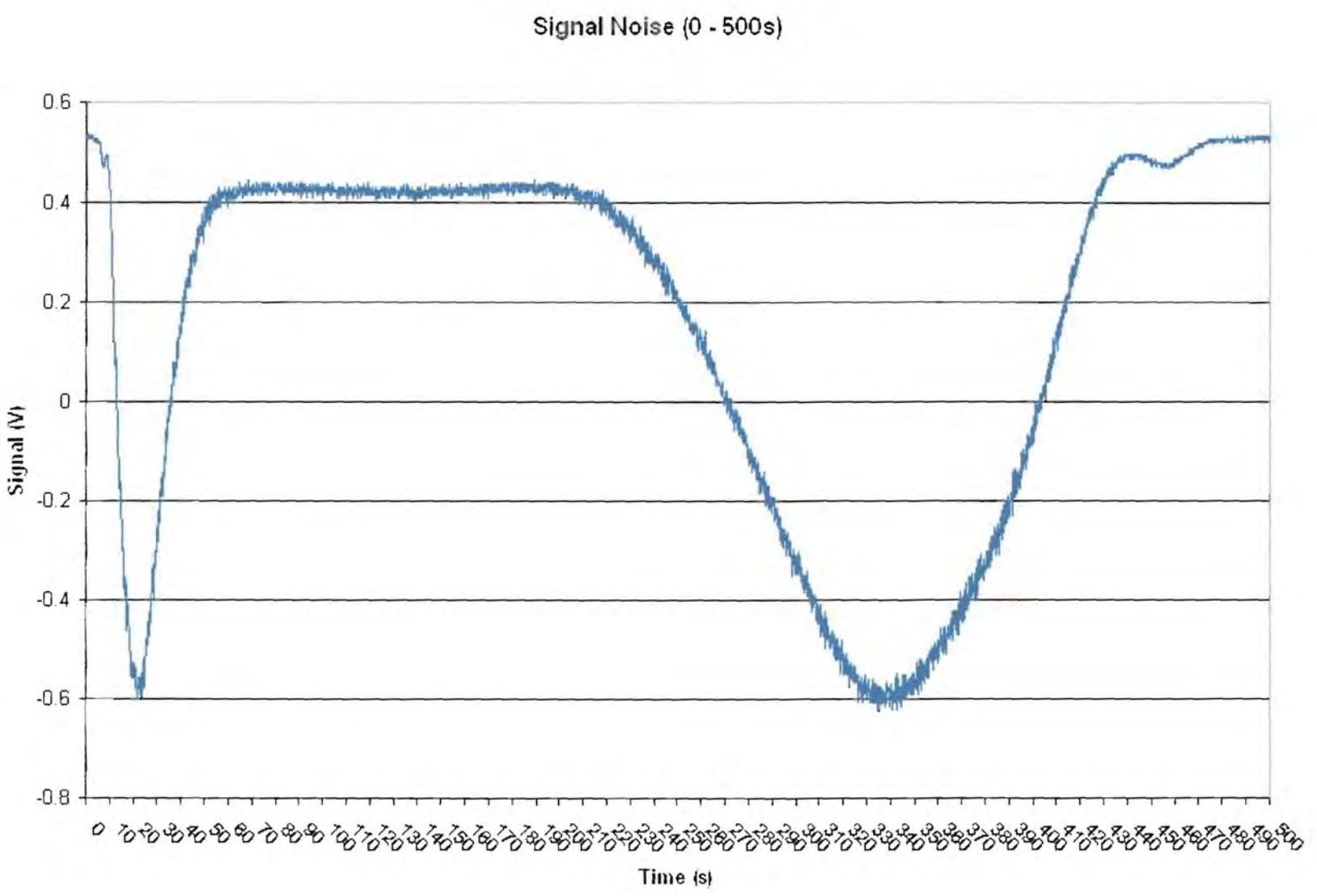

Disk 4 Run 4b Total
-0.085
-0.105
-0.125
-0.145
-0.165
-0.185
-0.205
-0.225
-0.245
-0.265
-0.285
-0.305
-0.325
-0.345
-0.365
-0.385
-0.405
-0.425
-0.445
-0.465
-0.485
-0.505
-0.525
Signal (V)
Time (s)

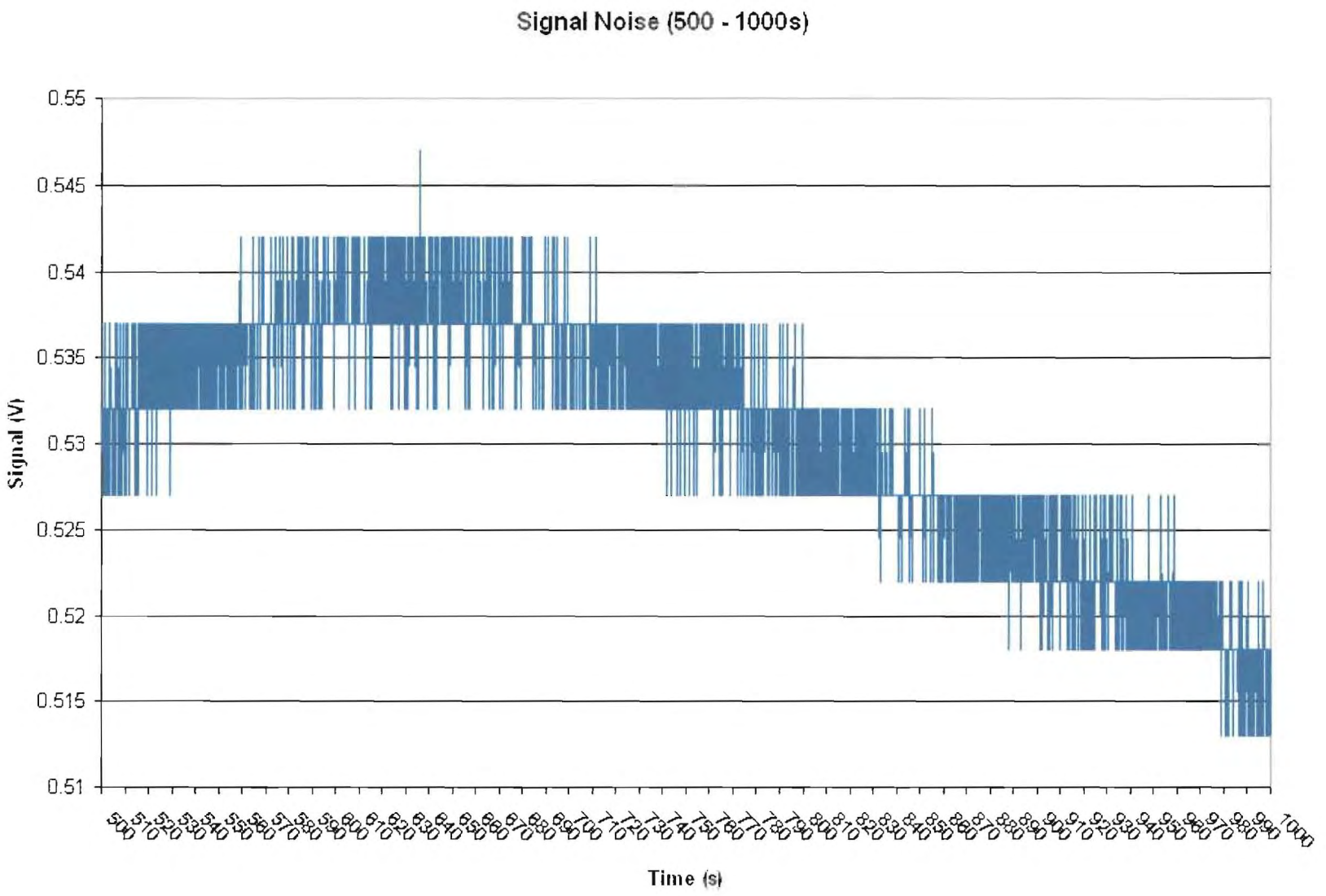

Disk 4 Run 5b Total
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-0.4
-0.45
-0.5
-0.55
Signal (V)
Time (s)

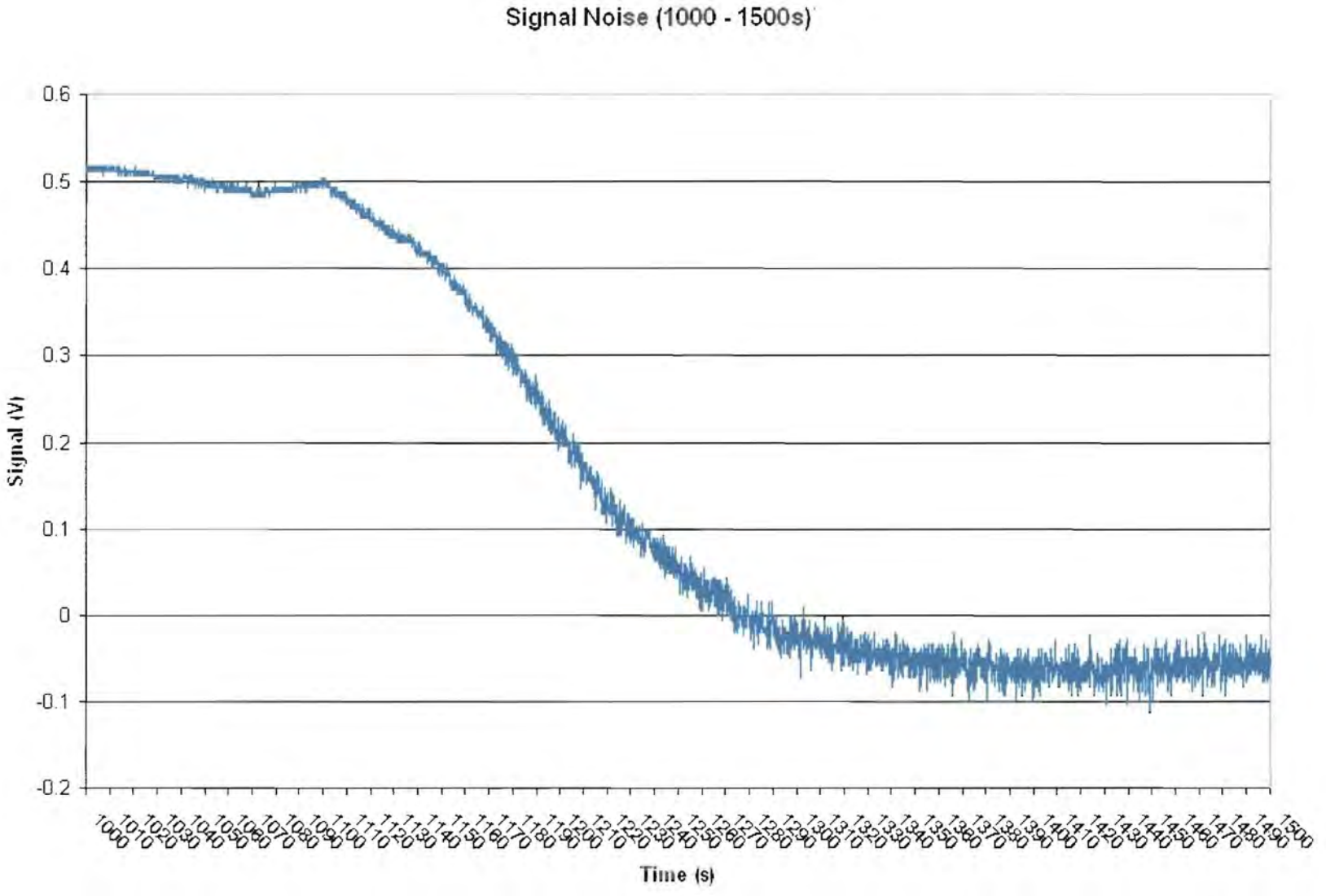

Disk 4 Run 5b
Signal (V)
-0.2
-0.21
-0.22
-0.23
-0.24
-0.25
-0.26
-0.27
-0.28
-0.29
-0.3
-0.31
-0.32
-0.33
-0.34
-0.35
-0.36
-0.37
-0.38
-0.39
-0.4
-0.41
-0.42
-0.43
-0.44
-0.45
-0.46
-0.47
-0.48
-0.49
-0.5
-0.51
-0.52
-0.53
-0.54
-0.55
Average: -0.24704 V
Midpoint: (-0.365 V, -0.5125 V) = 100.3 K
Average: -0.48305 V
Temperature (V)

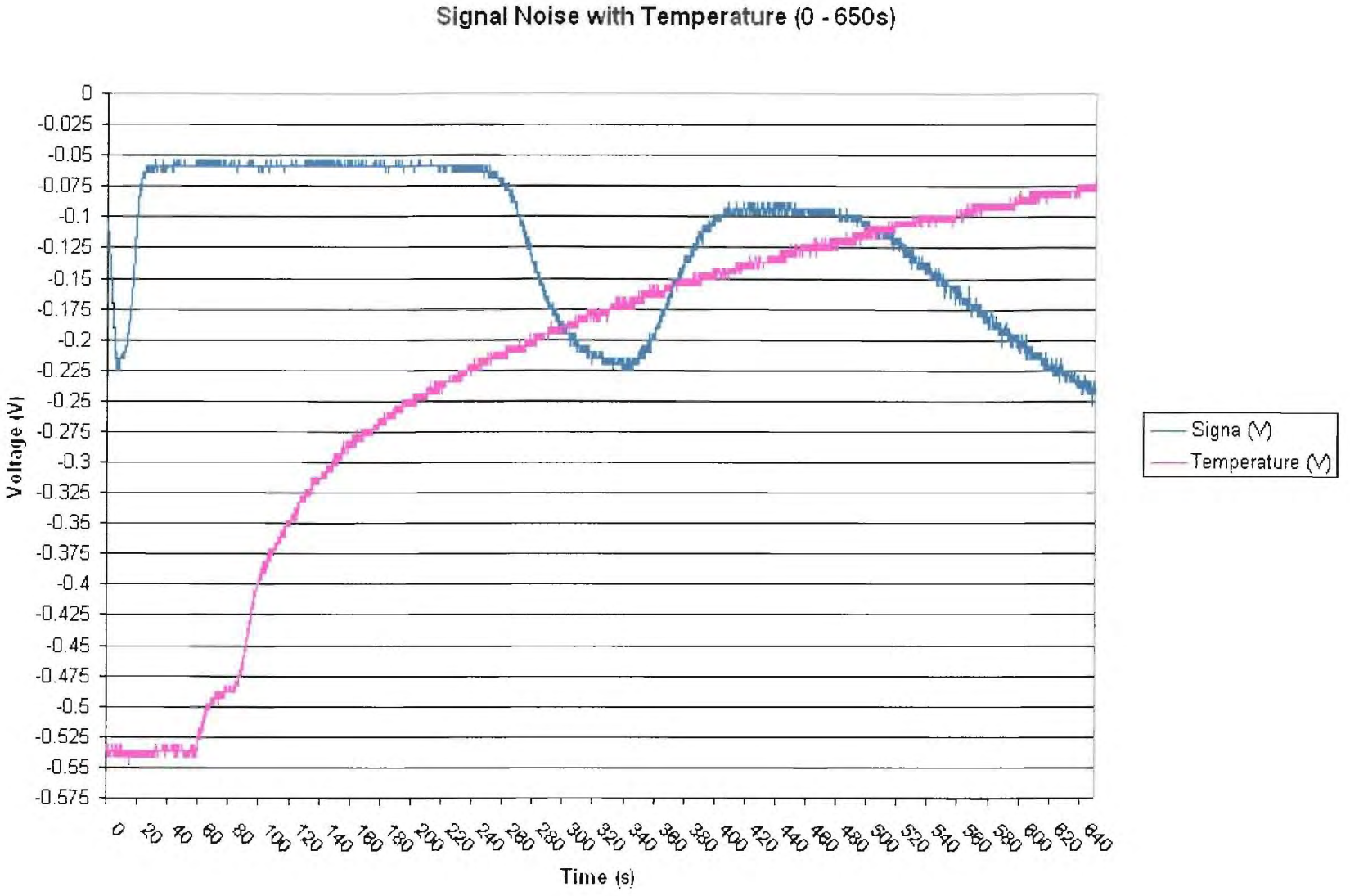

Disk 4 Run 6b Total
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-0.4
-0.45
-0.5
-0.55
-0.6
Signal (V)
Time (s)

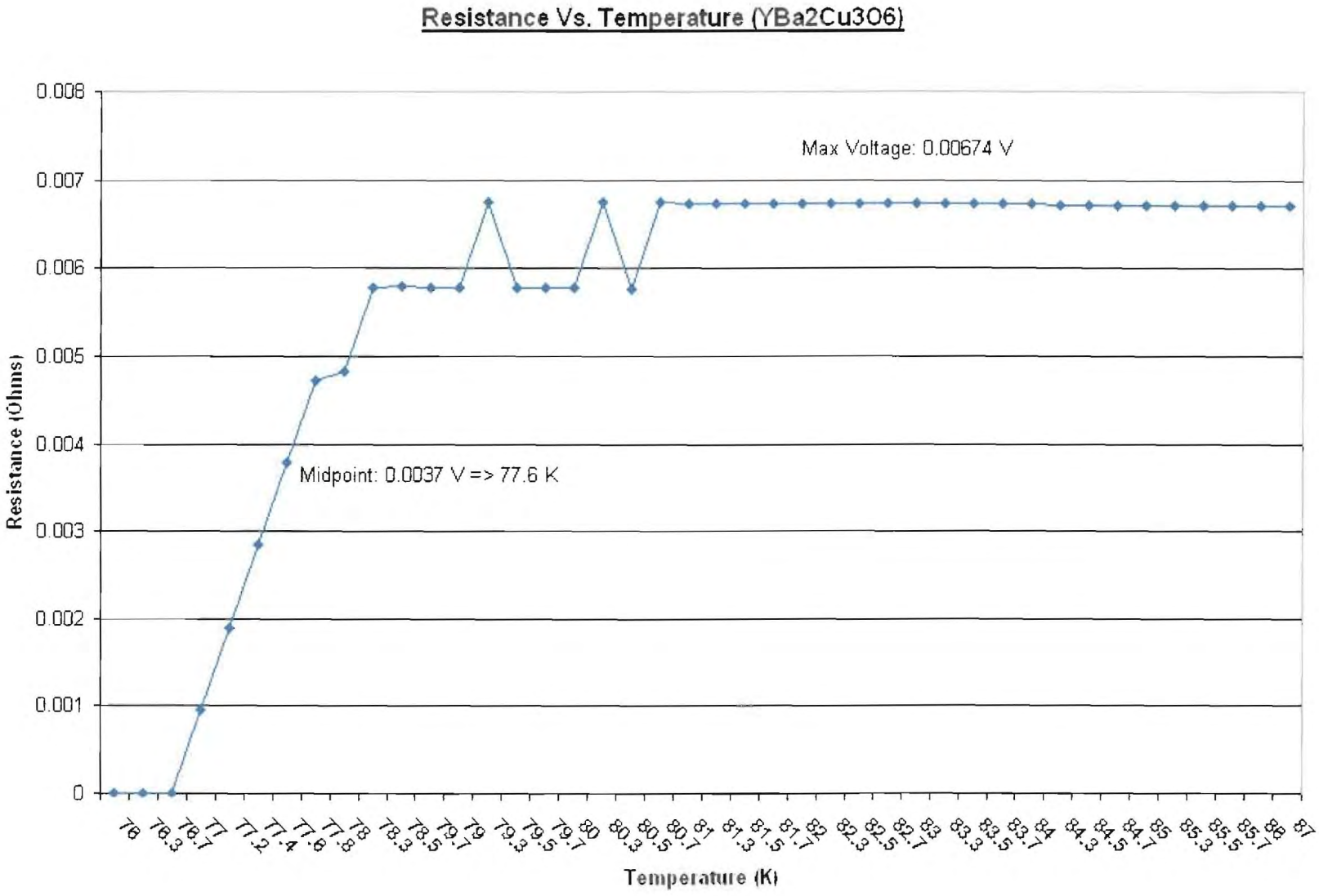

Disk 4 Run 6b
Signal (V)
-0.275
-0.285
-0.295
-0.305
-0.315
-0.325
-0.335
-0.345
-0.355
-0.365
-0.375
-0.385
-0.395
-0.405
-0.415
-0.425
-0.435
-0.445
-0.455
-0.465
-0.475
-0.485
-0.495
-0.505
-0.515
-0.525
-0.535
-0.545
-0.555
-0.565
-0.575
Average: -0.31983 V
Midpoint (-0.428825 V, -0.5175 V) = 97.5 K
Average: -0.53782 V
Temperature (V)

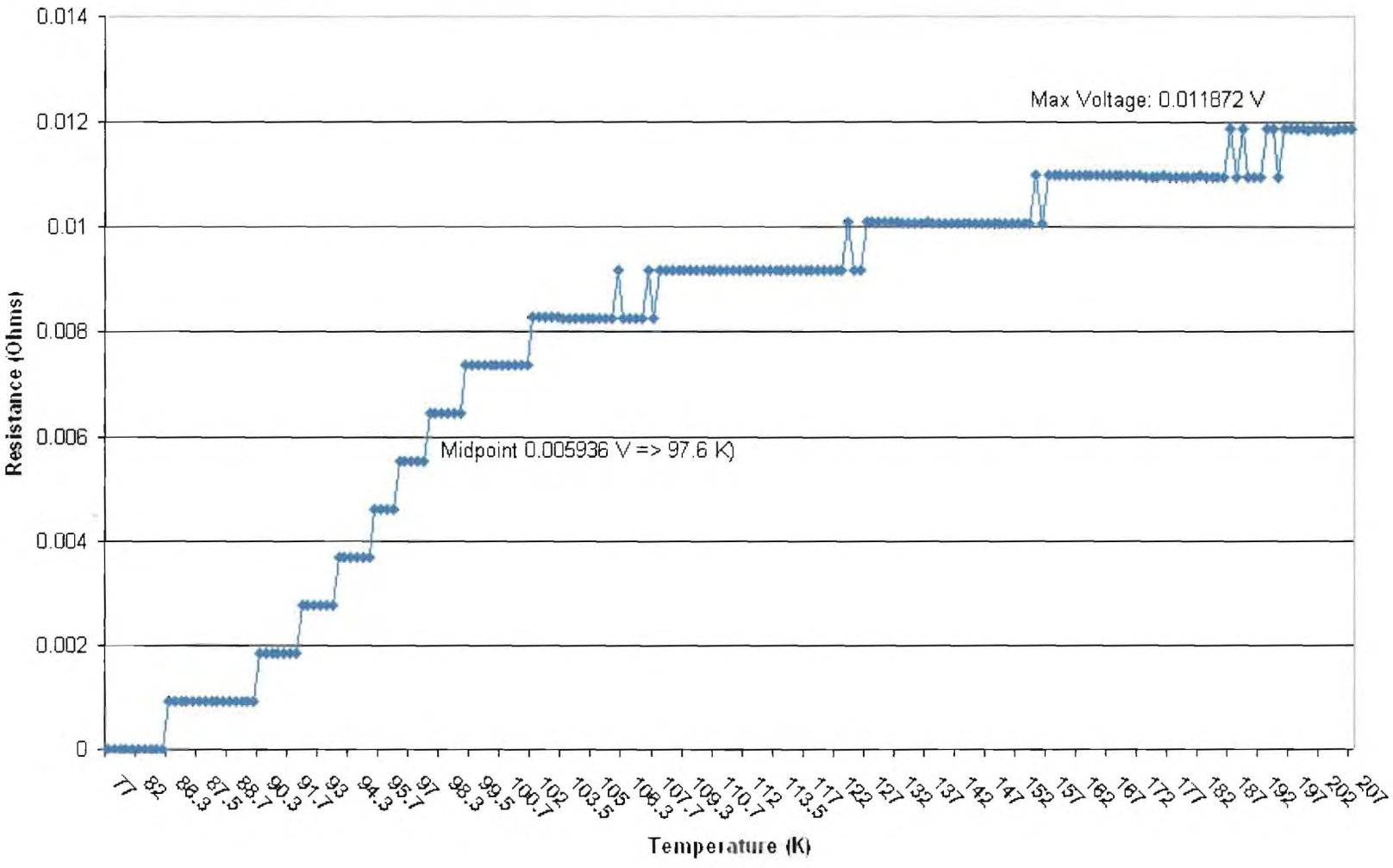

Signal Noise (0 - 500s)
Signal (V)
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220 230 240 250 260 270 280 290 300 310 320 330 340 350 360 370 380 390 400 410 420 430 440 450 460 470 480 490 500
Time (s)


Signal Noise (500 - 1000s)
Signal (V)
0.55
0.545
0.54
0.535
0.53
0.525
0.52
0.515
0.51
Time (s)


Signal Noise (1000 - 1500s)
Signal (V)
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
1000
1010
1020
1030
1040
1050
1060
1070
1080
1090
1100
1110
1120
1130
1140
1150
1160
1170
1180
1190
1200
1210
1220
1230
1240
1250
1260
1270
1280
1290
1300
1310
1320
1330
1340
1350
1360
1370
1380
1390
1400
1410
1420
1430
1440
1450
1460
1470
1480
1490
1500
Time (s)


Signal Noise with Temperature (0 - 650s)
Voltage (V)
0
-0.025
-0.05
-0.075
-0.1
-0.125
-0.15
-0.175
-0.2
-0.225
-0.25
-0.275
-0.3
-0.325
-0.35
-0.375
-0.4
-0.425
-0.45
-0.475
-0.5
-0.525
-0.55
-0.575
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640
Time (s)
Signa (V)
Temperature (V)


Resistance Vs. Temperature (YBa2Cu3O6)
Max Voltage: 0.00674 V
Midpoint: 0.0037 V => 77.6 K
Resistance (Ohms)
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
76
76.3
76.7
77
77.2
77.4
77.6
77.8
78
78.3
78.5
78.7
79
79.3
79.5
79.7
80
80.3
80.5
80.7
81
81.3
81.5
81.7
82
82.3
82.5
82.7
83
83.3
83.5
83.7
84
84.3
84.5
84.7
85
85.3
85.5
85.7
86
87
Temperature (K)


Resistance Vs. Temperature (Bi2Sr2CaCu3O9)
Max Voltage: 0.011872 V
Midpoint 0.005936 V => 97.6 K)
Resistance (Ohms)
Temperature (K)
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
77
82
86.3
87.5
88.7
90.3
91.7
93
94.3
95.7
97
98.3
99.5
100.7
102
103.5
105
106.3
107.7
109.3
110.7
112
113.5
117
122
127
132
137
142
147
152
157
162
167
172
177
182
187
192
197
202
207

SECTION V: Conclusions

Due to the large volume of data collected for each disk, only two sets of data were presented in this paper. Below is a chart of the critical temperatures derived from each of the graphs generated for each disk.

	Disk 1	Disk 2	Disk 3	Disk 4	Disk 5	Disk 6
Run 1	120	122.3	110	95	NT	126
Run 1b	x	x	118.9	125.5	132	x
Run 2	116.7	115.7	109	107.5	115.3	118.7
Run 2b	x	x	116.5	106.5	131.3	x
Run 3	113.3	116.7	109.5	104	113	107.5
Run 3b	x	x	110	107.25	127.7	x
Run 4	111	115.3	107.5	108.5	112	105
Run 4b	x	x	113	100.3	123.3	x
Run 5	111	114.7	107.5	86.7	113	88.3
Run 5b	x	x	116.6	100.3	125.5	
Run 6	x	x	x	NT	x	x
Run 6b	x	x	x	97.5	x	x
Average	114.4	116.94	111.85	103.55	121.4556	109.1
Standard Deviation	3.904485	3.083504	4.130443	9.794156	8.19407	14.40295

NT – No detectable transition
x – No data set

Fig. IV.a

Notice the wide range of average critical temperatures for each run. At first glance that would seem to indicate that there is something fundamentally wrong with the device, but further inspection proves otherwise. Disks 1 – 3 and disks 4 – 6 actually have significant differences between them. The major difference is that Colorado Superconductors produced disks 1 – 3, while students in the Modern Physics lab at Lycoming College produced disks 4 – 6. The disks manufactured by Colorado Superconductors had been stored in the lab for many years, without any markings to indicate what they are made of. At first, it was believed that they were actually composed of $YBa_2Cu_3O_7$, which has a critical temperature of about 92 K. Obviously, the temperatures calculated for disks 1 – 3 are much higher than that, but there is another superconducting material, $Bi_2Sr_2CaCu_2O_9$, that has a critical temperature of 110 K. Disks made out of the bismuth-based superconductor are available from Colorado Superconductors, and the Physics Department did purchase such disks in conjunction with the yttrium based ones. In the event that disks 1 – 3 were made out of $Bi_2Sr_2CaCu_2O_9$, then the corresponding error associated with the device for each disk is 4% for disk 1, 6.3% for disk 2, and 1.7% for disk 3. Unfortunately, disks 4 – 6, the ones made by Lycoming students, yielded quite different results. Those disks are made from a homemade superconductor kit that allows a student to make an $YBa_2Cu_3O_7$ disk, so there is no question as to the composition of disks 4 – 6. However, the calculated error for each disk is: 12.6% for disk 4, 32% for disk 5, and 18.6% for disk 6. By looking at both

the total run and transition graphs for each of those disks, it is easy to see why such erroneous and unfocused determinations of the critical temperature were calculated.

Many of the data runs do not have very definable transition periods as in the case of disk 4 runs 3b and 4b. The choice of transition periods in both those data sets was based on the apparent leveling-off of the graph towards the beginning and end of the periods. The apparent leveling could also just be noise inherent in the system, and the transitions in that case would be undetectable. All of the data taken for disk 4 that are not labeled "b" exemplify another problem in detecting the transition. Although in runs 1 – 5, it appears that there are transitions, those transitions occur either right before or right after a dramatic change in the signal. The behavior was investigated by taking two sets of data with no superconductor in the Experiment Coil. Instead, the coil was filled with liquid nitrogen and the signal from the device was measured over time. The three graphs (pg. 64 – 66) depicting the signal noise over a 25-minute time span show that there are indeed dips and rises within the noise generated by the device. A fourth graph (pg. 67) shows the second set of data that measures both the temperature inside the Experimental Coil and the signal generated by the device over a 650-second time period. The noise graphs reveal more evidence that the signal for the yttrium based disks might have been too small to be detected over the noise from the device.

A possible, but unlikely source of noise in the system could originate in the Reference Coil. During construction, the wire broke apart at the bottom of the first set of windings. It was promptly soldered back into place and protected extensively with gloss gel. While an extremely small joint, it could very easily distort the signal in the Reference Coil, which means that the Reference and Experiment Coils will never fully cancel one another when mixed in the preamplifier. That is not a big concern, since all that really matters is the sudden drop in the signal. However, there is a small possibility that the observed noise characteristics of the device could have something to do with the repaired joint.

It should be noted that all disks demonstrated the Meissner effect by floating a magnet before they were tested in the device, so the possibility of the disks not being in the superconducting state while data was taken can be ruled out. However, student made disks do have a tendency to not be fully superconducting throughout the entire disk. Thus, if a portion of the inside of such disks was the only part that had the ability to conduct without resistance, then the thermocouple on the outside surface of the disk would read a different temperature than the temperature of the superconducting part. The temperature would in fact be warmer then the temperature of the part in the superconducting state, which has been observed with the device. The temperature difference would probably not be as great as was measured, but it could still account for a few degrees of the disparity.

The construction method employed for making the ceramic disks could also be another source of disparity between different disks. It makes sense that the disks produced by Colorado Superconductors would all have about the same critical temperature, because their construction method was precisely controlled and easily repeatable. The construction method for the student made disks is not a constant repeatable process. To make such disks, students are required to subject a mix of yttrium, barium, and copper oxide to intense heat and ground into a fine powder before pressing the material into a disk. The purpose of the heating is to diffuse enough into the mixture

to achieve the desired crystal structure for a superconductor. However, the desired structure is not necessarily $YBa_2Cu_3O_7$. The chemical equation does not need to be stoichiometric, generally as long as there is more then six oxygen atoms in the structure then it will be superconducting at low enough temperatures. Different grinding, heating and pressing methods will lead to differences in the amount of oxygen in the crystal structure of each disk. Thus each yttrium based disk could potentially have a different critical temperature associated with it. The differences observed between disks 4 – 6 could be explained by the slightly different construction methods of students.

To round out the study, an experiment using the Modern Physics lab's current method of measuring critical temperatures was carried out. The current method entails using disks that already have wires for measuring resistance built into them. The results from the mini investigation can be found on pages 68 – 69, titled "Resistance Vs. Temperature" with the compound name attached. Both $YBa_2Cu_3O_7$ and $Bi_2Sr_2CaCu_2O_9$ were studied, with results comparable to, if not worse than the results from the non-intrusive device. A critical temperature of 77.6 K for $YBa_2Cu_3O_7$ and a critical temperature of 97.6 K for $Bi_2Sr_2CaCu_2O_9$ were measured by the four-point probe. The error in this case could be due to an uncalibrated thermocouple, or a non-zeroed digital Multimeter. There were no procedures in the Modern Physics lab manual for calibrating the thermocouple, or zeroing the DMM, so no calibrations were carried out. However, the temperature data from the experiment is indicative of a correctly working thermocouple, since the data begins at 77 K, the boiling point of liquid nitrogen. The DMM also appeared to be reading correct values at the start of the experiment, but because no calibrations of these devices were carried out, it is impossible to state that they are not a source of error. In any case, the one set of data from the four-point probe is not enough to substantiate any claims that the new device is a more accurate means of measuring the critical temperature of superconductors. The new device does have one advantage over the four-point probe; it allows students the opportunity to study the properties of the superconductors they make, provided its noise characteristics can be worked out.

Upon completion of the investigations, there have been some breakthroughs and some disappointments. The device did detect transitions in superconducting disks, but how accurately it actually measures the critical temperature is still undetermined. More studies on the noise of the signal need to be done to resolve what could be causing such noise, and whether or not it is possible to isolate and remove the noise from the data. A resonance within the system that is set up by the frequency of the original signal generated by the function generator in the SPLIA1-A should be the first theory to be tested concerning the noise of the system. Such a signal would repeat over time, which was observed in the signal noise graphs as successive dips in the signal voltage.

However, the dips do not occur in a periodic form, as a resonance is most likely to do. The change in resistivity of the copper wiring as the temperature increases could have an effect on the periodic nature of the signal. An in depth study of the baseline output of the device should be conducted for all possible frequencies and amplitudes of the signal generator. Some possible research topics that could be covered are: the noise as a function of the temperature inside the device, noise due to differences in temperature of different parts of the system, and electronic interference from the surrounding environment. Such an investigation could probably be conducted in a semester long

research topics or independent study course by a future interested student. This investigation could lead to two other projects that future students could study.

The first would be to use the device to study the transition between diamagnetism and paramagnetism in magnetic materials. Because the device essentially measures the Meissner effect, which is really diamagnetism, it is well suited for such studies, provided they are done at low (liquid nitrogen) temperatures.

Another study could be conducted on the feasibility of using the Physics Department's electromagnet to measure the critical fields of superconductors made in the Modern Physics lab. Critical temperatures and fields are two of the most important properties that physicists measure in superconductors, adding a component on critical fields would greatly increase the ability for students to learn about and appreciate the physics of superconductors.

A direct consequence of this investigation has been the development of two lab experiments for use in the Modern Physics course at Lycoming College: a lab on the Franck-Hertz Experiment (Appendix A) to demonstrate the physics of quantum mechanics, and a lab on determining the critical temperature of laboratory made superconductors using a non-intrusive method (Appendix B). Hopefully those experiments will bolster the inquisitive nature of future students and instill in them a sense of wonder at how amazing the world around them can be.

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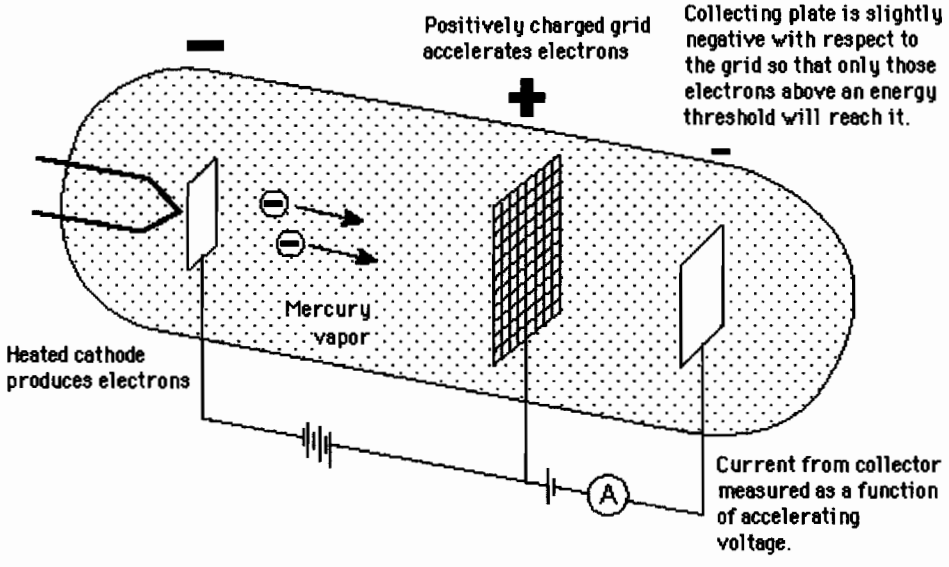
APPENDIX A:
Franck-Hertz Experiment

Franck – Hertz Experiment

INTRODUCTION:

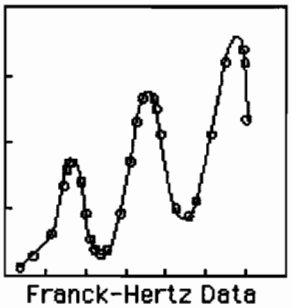
In October 1900 the physicist Max Planck suggested that when an oscillator emits radiant energy, it does so in the form of electromagnetic waves and only in discrete amounts, which he called quanta. This new theory led to what is now called Quantum Mechanics, and, would throw the world of classical physics into such upheaval, that even Planck himself was uncomfortable with his own idea. Many scientists did not take Planck's work seriously until 1905 when Albert Einstein used Planck's theory to explain the photoelectric effect. After that, the floodgates were open and many other phenomena were explained using quantum mechanics. James Franck and Gustav Hertz conducted one such experiment in 1914 which history has appropriately named: The Franck – Hertz Experiment.

In their investigation, electrons were accelerated by a voltage toward a positively charged grid in a glass tube filled with mercury vapor. By adjusting the potential between the cathode and the positively charged grid, Franck and Hertz were able to increase the energy of the electrons. Past the grid was a collection plate held at a small negative voltage with respect to the grid. The electrons would then be accelerated through the grid where they would subsequently collide with mercury atoms in the tube. After the collision the electrons would be attracted to the positively charged collection plate, however, if the energy left in an electron after a collision was too small, the slightly more positive grid would attract the electrons back away from the collector plate.



Since the mercury atoms could theoretically only hold quantized amounts of energy, the electrons would only transfer specific energies to the mercury atoms. Thus, if the energy of an accelerated electron were the same as the excitation energy of a mercury atom, then virtually all of the electron's energy would be transferred to the mercury

(assuming completely elastic collisions). That would not leave enough energy for the electron to reach the collector and it would be attracted back to the grid. As a result, the current registered by the collector plate would drop at specific intervals corresponding to the excitation states of the mercury atoms. When Franck and Hertz plotted the accelerating potential versus the current from the collection plate their data showed exactly that, helping to further validate Planck's revolutionary hypothesis.



OPERATION:

These instructions are adapted from the instruction manual for the Pacific Science Supplies' Franck – Hertz Apparatus Model P67103 that is used in this experiment. The device uses an argon tube instead of mercury in order to eliminate heating of the tube, which reduces the time over which the experiment needs to be performed in. Data can be taken manually or directed to an oscilloscope or a computer for display.

The controls on the panel of the device are shown in Fig.1.

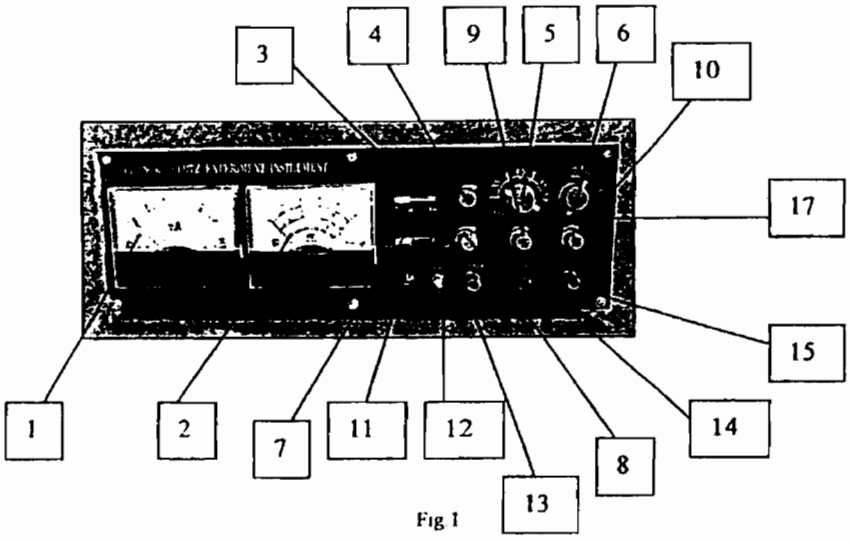


Fig 1

1. Rheometer
2. Voltmeter
3. Manual-Auto switch
4. Scan knob
5. Filament Voltage selector
6. Current Multiple selector
7. Voltage Stepper
8. 1.3 ~ 5V adjust knob (Ug2k)
9. 1.3 ~ 5V adjust knob (Ug2A)
10. 0 ~ 100V adjust knob
11. Indicator
12. Power switch
13. Y- output terminal
14. Ground
15. X-output terminal
16. Power wire
17. Observe hole

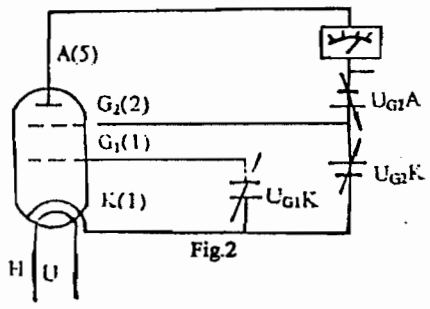


Fig.2

1) Switch on the power. The indicator will flash.

2) Turn the "Manual – Auto" switch to "Manual", rotate the scan knob counter – clockwise until it stops, and turn "Filament Voltage Selector to 3.5V, "Current Multiple" selector to 10^{-7}.

3) Turn the "Voltage Stepper" to 1.3 ~ 5V, and rotate 1.3 ~ 5V adjust knob until the voltmeter reads 1.5V. This sets U_{G1K} = 1.5V.

4) Turn the "Voltage Stepper" to 1.3 ~ 15V and rotate 1.3 ~ 15V adjust knob until the voltmeter reads 7.5V. This sets U_{G2A} = 7.5V (rejecting voltage).

5) Turn the "Voltage Stepper" to 0 ~ 100V, and rotate 0 ~ 100V adjust knob until the voltmeter reads 7.5V. This sets U_{G2K} = 7.5V (accelerating voltage).

   When you have finished steps 2 – 5, with U_H = 3.5V (filament voltage), U_{G1K} = 1.5V (voltage between the second grid and the collector anode) you are ready to do the experiment. These are the suggested voltages for the experiment; you can also do the experiment using the parameters marked on the argon tube.

6) If the Franck – Hertz tube is not in the apparatus, remove the cover of the instrument and place the tube in the lamp socket. Replace the cover and turn on the power. The indicator will flash.

7) Preheat the tube for 3 minutes before the experiment.

8) Rotate the 0 ~ 100V adjust knob while observing the variation in the ammeter and voltmeter readings. With the increase of U_{G2K} (accelerating voltage), the ammeter's reading appears to peak and vary periodically. Record the corresponding voltage and current.

9) For using an oscilloscope or computer in this experiment, turn the "Manual – Auto" switch to "Auto", and connect the instrument's Y, ground, and X socket to the Y, ground, and X socket of an oscilloscope or computer. If using an oscilloscope, switch the scanning range of it to "external X". Switch on the power of the oscilloscope and adjust the Y and X shift to make the scan baseline on the bottom of the screen, and adjust the X gain to make the scan baseline 10

grids. Rotate the scanning knob of the oscilloscope and observe the waveform on its screen. Adjust the "Y gain" and "X gain" of the oscilloscope's attenuation to make the waveform clear and Y amplitude moderate. Rotate scanning potentiometer clockwise to end, set the scan voltage to 50V, and measure the horizontal distance of two consecutive crests (count the grids). Multiply the distance by 5V/grid to obtain the value of an argon atom's first excitation potential.

Caution

1) During the experiment, pay attention to the output current indicator when the voltage is over 60V. If the Ammeter's reading increases suddenly, decrease the voltage at once to avoid damage to the tube.

2) If you want to change the value of U_{G1K}, U_{G2A}, and U_H during the experiment, rotate the 0 ~ 100V adjust knob counter – clockwise to end, before making the changes.

3) The filament voltage of this instrument is 3V, 3.5V, 4V, 4.5V, 5V, 5.5V, and 6.3V. You can do the experiment with these filament voltages. If the waveform is skewed (that means the anode output current is too strong and causes the amplifier to distort), the filament voltage should be decreased.

Specifications

1. Voltage supplied to Franck- Hertz tube.

U_{G1K}	1.3 ~ 5V
U_{G2A} (rejecting voltage)	1.3 ~ 15V
U_{G2K} point- measure observe	0 ~ 100V
Sawtooth wave on oscilloscope	0 ~ 50V
U_H (filament voltage)	AC: 3V,3.5V, 4V, 4.5V, 5V, 5.5V, 6.3V.

2. Parameter about sawtooth wave.

Scanning Voltage	0 ~ 50V
Scanning Frequency	115 ± 20Hz
Voltage amplitude of scanning output	≤ 1V

3. Low-current Measuring range

10^{-9} ~ 10^{-6} A(4 steps)

4. Observable numbers of spectrum amplitude

Point – measuring	≥5
Observe on universal oscilloscope	≥2

5. Operating condition

Ambient temperature	-10 ~ 40°C
Relative humidity	≤85%(40°C)
Operating power	AC 220V ± 22V, 50Hz
Preheating time	≤5 min
Continuous operating time	8 hours
Rated input power	≤15W
Dimensions	l X b X h_{mm}: 400 X 230 X 130

APPENDIX B:
Measuring Critical Temperatures of High Temperature Superconductors

Measuring the Critical Temperature of High Temperature Superconductors

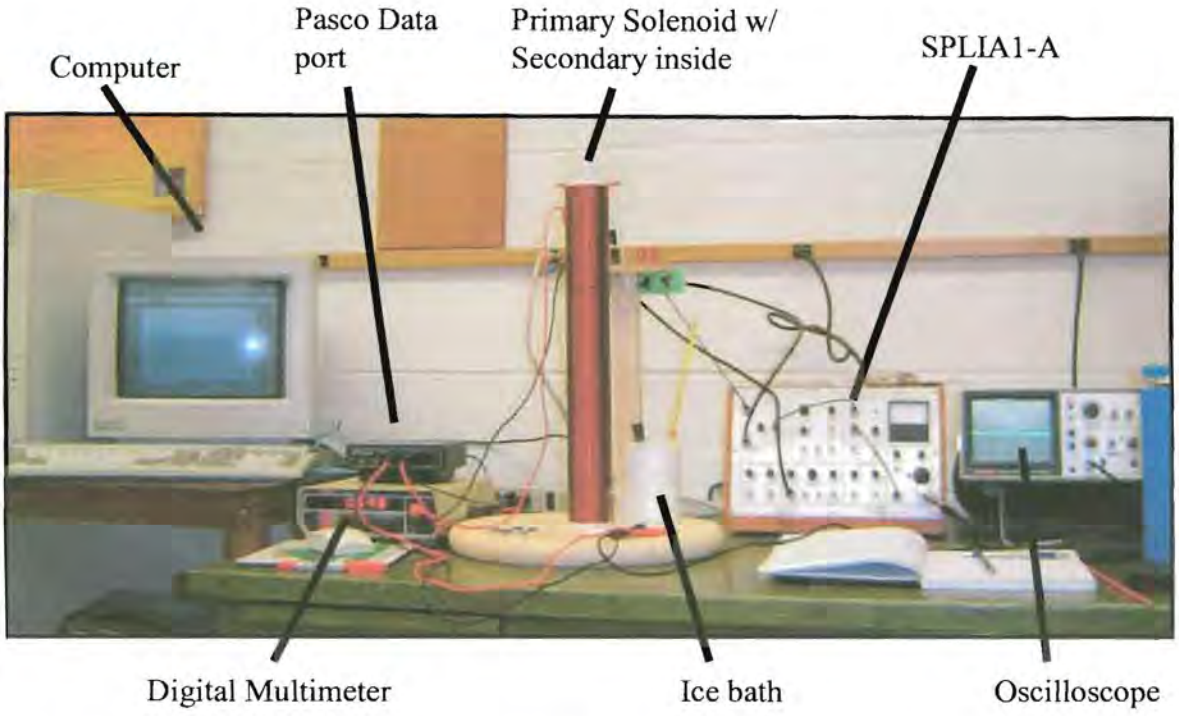
INTRODUCTION:

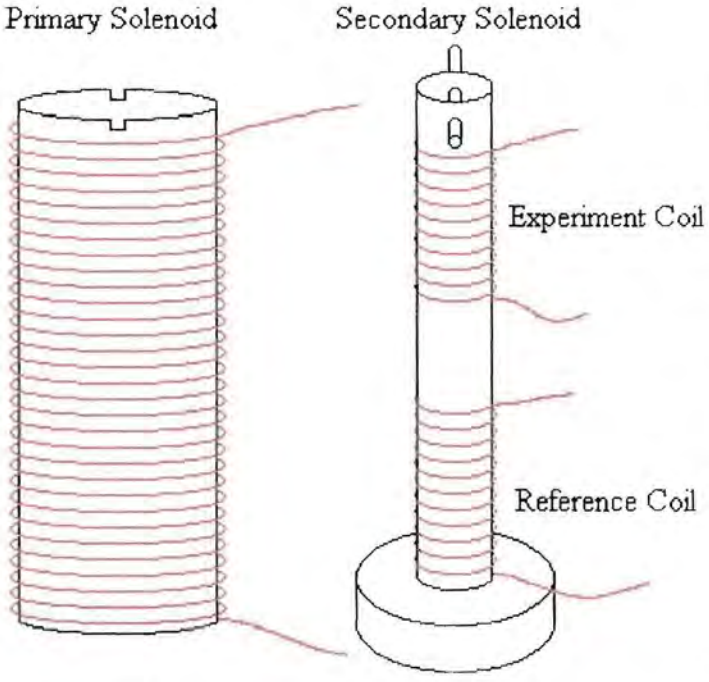
In 1911, Heike Kamerlingh Onnes began experiments to test the electrical properties of various metals at low temperatures using liquid helium (~ 4 K). It was his hypothesis that in a lower state of energy, there would be less resistance to the flow of electricity in metals. Onnes had expected to find resistance drop exponentially with temperature and then eventually level off to a limiting resistance that would never be able to be overcome. He observed that indeed resistance dropped as a function of temperature but he also observed something unexpected and astonishing. To Onnes' surprise, he found that around 4 K mercury completely lost all resistance as did various other metals. He even formed a ring out of mercury, submerged it in liquid helium, and passed a current through it. Onnes then allowed it to sit like that, while making sure there was always plenty of coolant around the mercury. After a year, he measured the current in the ring and found that it had persisted virtually undiminished! Often scientists will talk about physical phenomena approaching zero, or being negligible, but very rarely do they find something as remarkable as Onnes had. Onnes termed his new discovery superconductivity and described the temperature at which a superconductor lost resistivity as the critical temperature (T_c). The scientific community realized the importance of Onnes' discovery and quickly rushed to find materials with higher critical temperatures. Liquid helium is relatively expensive to produce so the prospect of finding a commercially viable superconductor that had a higher T_c was very lucrative.

All the known metals were subjected to test after test and when they were exhausted, combinations of different metals were tested to see if any would yield significant results. Unfortunately, progress towards raising the critical temperature stalled for 60 years before IBM researchers Alex Müller and Georg Bednorz had constructed a ceramic superconductor with a critical temperature of 30 K. Soon after, in 1987, Paul Chu and a research team at the University of Houston reported developing a material that would become superconducting at 98 K, which could be cooled using liquid nitrogen, a safer and more readily available cryogen. With the development of these new "high temperature" superconductors, the field of solid-state physics was rejuvenated with the hope of inventing materials that would become superconducting at room temperatures.

Setup:

The apparatus used for measuring the critical temperatures of superconducting pellets (most likely made out of $YBa_2Cu_3O_7$) has many different parts associated with it. One should become familiar with the names and the procedures of each device, especially the SPLIA1-A and the Data Studio computer program.





The signal generator on the SPLIA1-A is connected to the primary solenoid. The output from experiment coil is connected to the A jack and the reference coil connects to the –B jack on the preamplifier on the SPLIA1-A. The preamplifier combines the signals and the resulting signal is sent to the amplitude detector where the signal is rectified and passed on to the low-pass filter/amplifier. The filter averages the rectified signal and returns a DC voltage that is sent to the Pasco Science Workshop 500 Interface and the oscilloscope. (The oscilloscope is only used as another means of observing what is going on, other than Data Studio and the built-in gage on the low-pass filter/amplifier.) When a signal is passed through the primary, it induces a signal in the coils in the secondary solenoid. When the secondary is empty, the reference signal and the experiment signal will be the same, and thus cancel out when mixed by the preamplifier.

When a superconducting disk is placed in the experiment coil and cooled, then the Meissner effect will warp the magnetic field inside the experiment coil and alter the

experiment signal. Thus a signal can be passed on to the amplitude detector and a voltage registered by the Pasco 500. Because the amplitude detector rectifies signals negatively, the registered DC voltage will be negative. A thermocouple connected to the superconducting disk will be outputting voltages to a digital Multimeter (DMM) while the device outputs a signal voltage. Both voltages are fed into Data Studio, which records them simultaneously. The simultaneous recording of the two signals is the key for determining the critical temperature of the disk.

Procedure:

1) Connect the thermocouple to the DMM and the DMM to the Pasco 500 with banana plugs. Make sure the DMM is set to (GET THIS SETTING).

2) Calibrate the thermocouple by placing both junctions in test tubes filled with mercury, and then placing the test tubes ice baths. Allow the voltage on the (DMM) to reach an equilibrium, then adjust the zero knob on the DMM until it reads zero.

3) Remove the upper junction from its ice bath and test tube. Carefully return all mercury from the test tube to its original container.

4) Zero the oscilloscope.

5) Set the gain on the preamplifier to 50V. Then set the gain on both the amplitude detector and low-pass filter/amplifier to 2V. Also on the low-pass filter/amplifier are the time constant and the dB/oct switch. These should be set to 0.1 and 12 respectively.

6) Use the DC-offset on the low-pass filter/amplifier to zero any signal in the device. The oscilloscope or the dial on the filter/amplifier are useful for doing this.

7) Secure the free junction on the thermocouple to the disk that is to be studied. Place the disk inside the experiment coil so that it lays flat on the styrofoam base. A plastic rod can be used to press the disk flat.

8) Set up Data Studio to take data: make sure that both volt sources are connected and register as active. Set the sample time to be 0.1s. Bring up the "Graph" option for the best interpretation of the data.

9) Add liquid nitrogen to the secondary so that it completely covers the disk.

10) When most of the nitrogen has boiled away, click the start button in Data Studio to begin taking data.

11) When the signal voltage shows the transition, allow the experiment to run for around 30 more seconds. If there is no discernable transition, wait until the

thermocouple voltage reads at least -0.425 V ~ 140 K before stopping the run. This is because the critical temperature will never be that high for $YBa_2Cu_3O_7$ or $Bi_2Sr_2CaCu_2O_9$.

12) Click the stop button to stop taking data.

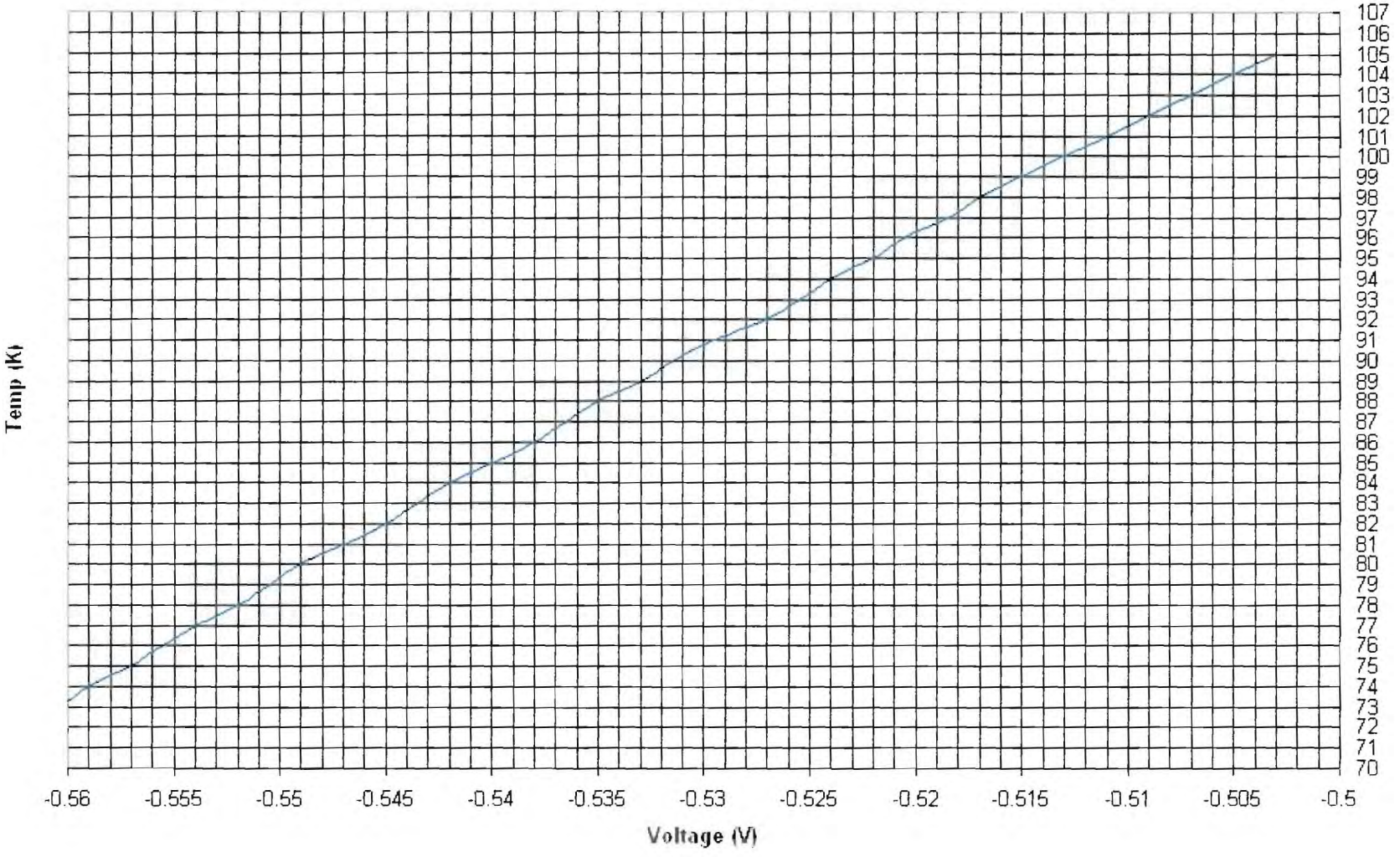
13) Use the "Table" feature to cut and paste data from Data Studio into Microsoft Excel for analysis.

14) The critical temperature can be determined by finding the time at the middle of the transition. That time will have a corresponding voltage from the thermocouple. Use the calibration curve for the thermocouple on the next page to find the critical temperature.

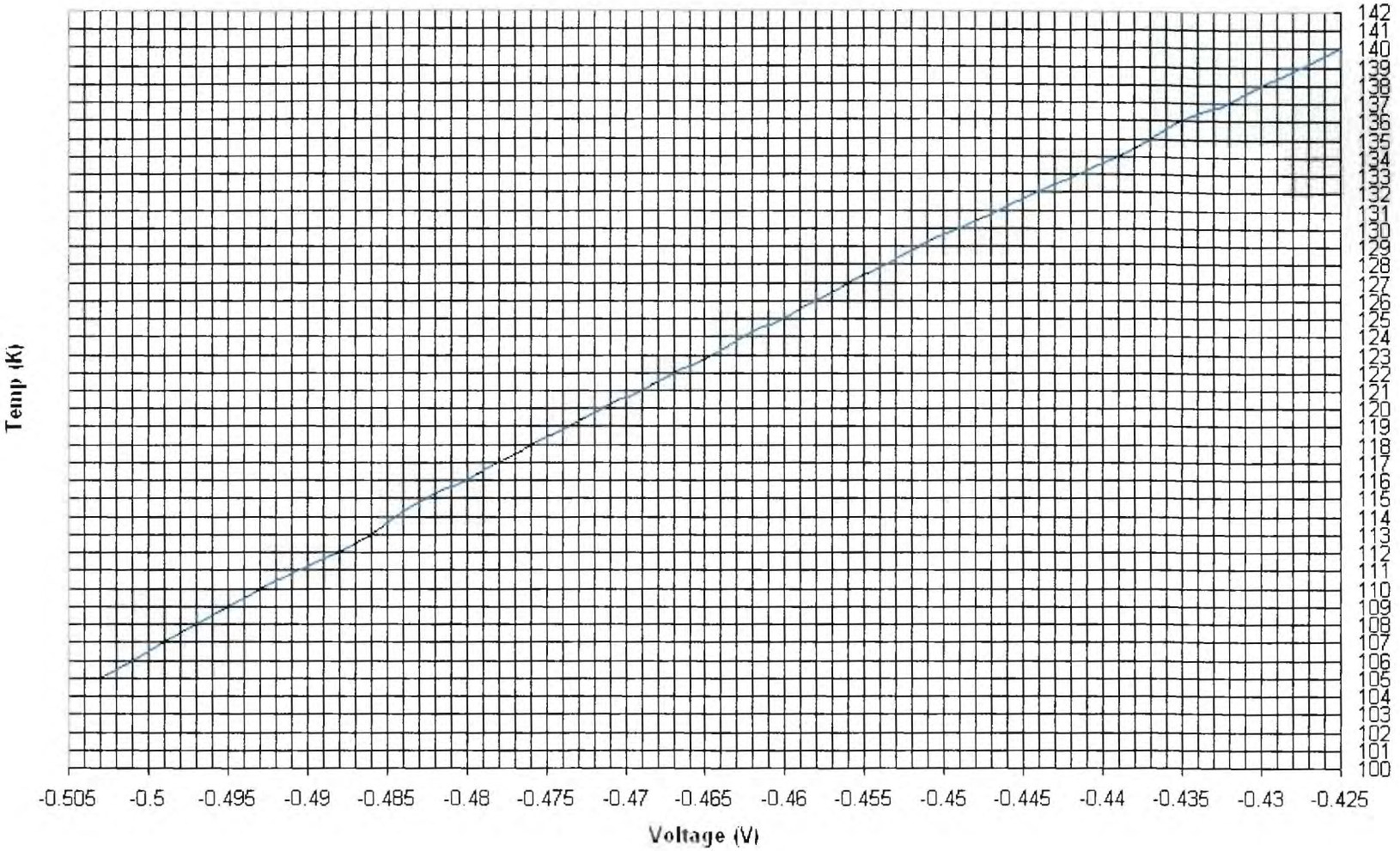
15) Perform as many runs as deemed necessary. In general, the more runs, the more accurate the determined critical temperature will be.

*For more details on the physics of superconductors and how the T_c measuring device works see: An Experimental Investigation of High Temperature Superconductors by Robert Benoit, a copy can be found in the Snowden Library

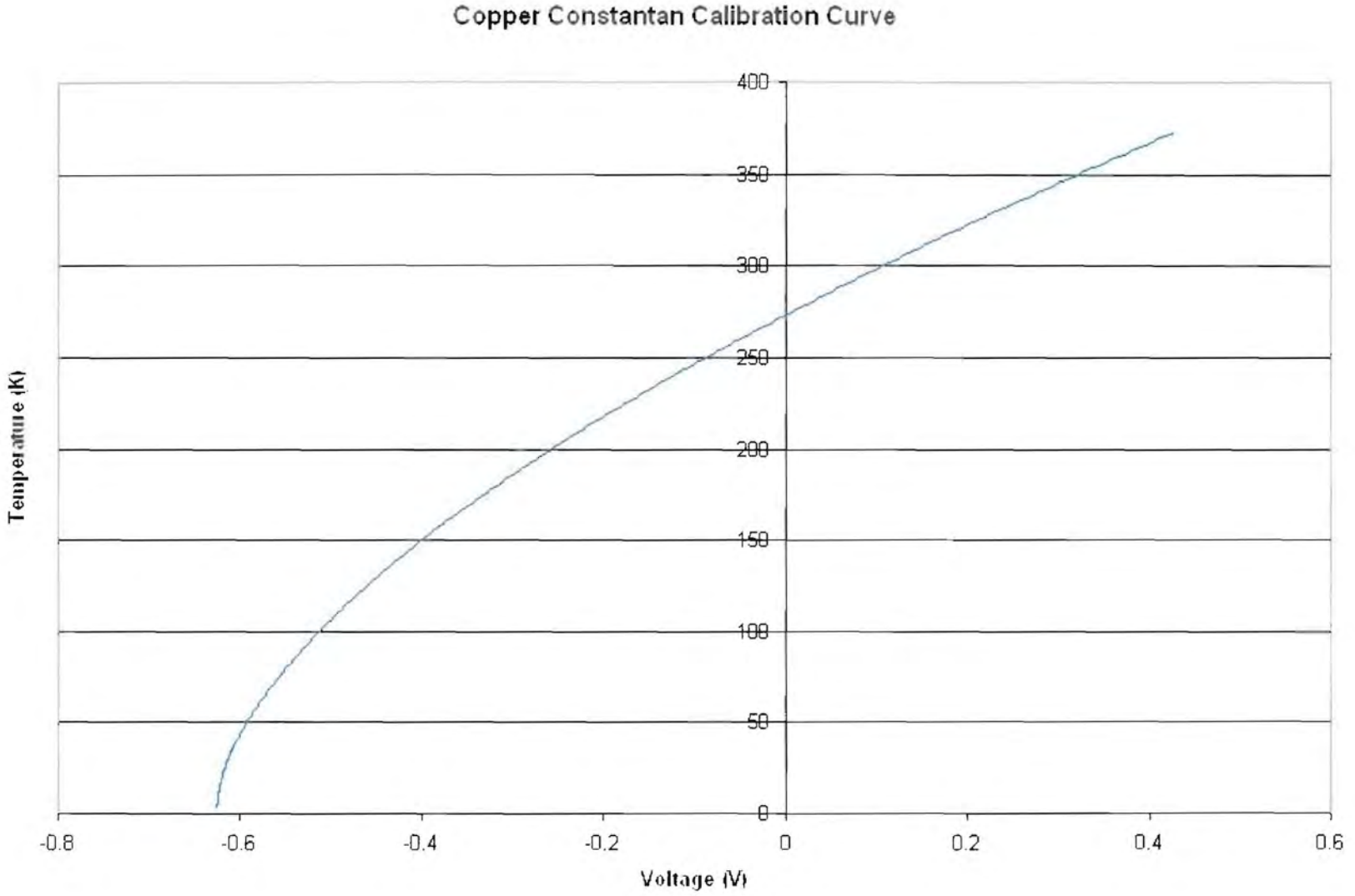
70 - 140 K


Temp (K)
107
106
105
104
103
102
101
100
99
98
97
96
95
94
93
92
91
90
89
88
87
86
85
84
83
82
81
80
79
78
77
76
75
74
73
72
71
70
-0.56
-0.555
-0.55
-0.545
-0.54
-0.535
-0.53
-0.525
-0.52
-0.515
-0.51
-0.505
-0.5
Voltage (V)

105 - 140 K


Temp (K)
142
141
140
139
138
137
136
135
134
133
132
131
130
129
128
127
126
125
124
123
122
121
120
119
118
117
116
115
114
113
112
111
110
109
108
107
106
105
104
103
102
101
100
-0.505
-0.5
-0.495
-0.49
-0.485
-0.48
-0.475
-0.47
-0.465
-0.46
-0.455
-0.45
-0.445
-0.44
-0.435
-0.43
-0.425
Voltage (V)

APPENDIX C:
Calibration Curve for Copper-Constantan Thermocouple


Copper Constantan Calibration Curve
Temperature (K)
400
350
300
250
200
150
100
50
0
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
Voltage (V)