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Math 234W

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Formal Writing Portfolio

Mathematical Autobiography

Mathematical education has proven to be exciting, tedious, and rewarding, and it is the

leading contender in becoming my future occupation. Because I am a mathematics major

certifying in secondary education, I relate my experiences of mathematics closely to the

teachers who taught me everything from addition to triple integration. I developed a love and

appreciation for mathematics due to the passion with which many of my teachers taught it. The

first time the excitement of mathematics grasped me was the second grade. The feeling has yet

to leave.

In second grade, my teacher, Mrs. Allison, taught the basics of math including a way to

learn multiplication with the use of plastic cubes you link together. I enjoyed playing with blocks,

and working with cubes was almost the same thing. To me, "math time" sounded more like

"block time," and I was hooked. In third grade, I was placed in the gifted program. I was in the

program for the rest of elementary school during which I learned a lot of algebra. My teacher,

Mrs. Bauman, told my parents I had a natural mathematical ability.

Everything ran smoothly through middle school. Sixth grade's basic algebra, seventh

grade's geometry, eighth grade's algebra II—the concepts were all interesting yet simple. My

freshmen year of high school proved to be more difficult. I was intimidated by taking a math

course with upperclassmen. Mr. Gilmore, the functions, statistics, and trigonometry teacher,

did not help. He was a large man who believed in teaching students to learn material on their

own. He gave little guidance, explaining that if there was ever a question, the answer could be found by using the book. His exams were difficult and always included one nearly impossible problem that had nothing to do with what we had learned. Mr. Gilmore, who was my transition to upper-level mathematics courses in high school, was giving me a reason not to care for the subject. Luckily, Mr. Kramer, my tenth grade trigonometry teacher, was able to make mathematics enjoyable again. He was a soft-spoken, funny, friendly man who created a relaxed and amusing atmosphere in which to do trigonometry. In his class SOH-CAH-TOA was an Indian chant used to help everyone remember how to calculate sine, cosine, and tangent. I had Mr. Kramer my junior year for calculus as well. With Mr. Kramer's guidance, I conquered simple derivatives and summation. By the end of my junior year, I knew I was destined for a future in mathematics; I just wasn't sure what.

As a senior, I took Advanced Placement (AP) calculus. It was the most exciting course I had ever taken in high school. The work was challenging, time consuming, and yet invigorating. Mrs. K. Miller, my teacher, was spectacular. She had a dedication to her students I had never experienced before. Due to my passion for calculus and her aptitude for teaching, I received a 5 on the AP calculus exam. Mrs. K. Miller also inspired me to become a teacher. That led me to Lycoming College and the academic path I am pursuing.

Now in my sophomore year at college, I've learned much about derivatives, integration, series, and other calculus related topics. In the fall of 2006, I took calculus II. My first mathematics professor at Lycoming College, Dr. Yin, taught the course and made my love for calculus run deeper. In the fall of 2007, I took calculus III with Dr. Sprechini. These two courses confirmed my desire to become not only a teacher, but a *calculus* teacher.

I've always enjoyed mathematics and plan on doing so for the rest of my life. After I graduate, I'd like to teach students the joy of mathematics as well. Perhaps, years from now, some student will include my name in a paper such as this, which would be quite an honor.

Mathematician Biography: Rene Descartes

Rene Descartes was not solely a mathematician. He was, above all else, a philosopher who was quite accomplished in mathematical techniques involving algebra and geometry. He had not been exposed to mathematics until his last two years of school at the Jesuit college of Henri IV in La Flèche. Though he showed great aptitude for mathematics, he continued his education at Poitiers, earning his baccalaureate and licentiate degrees in Law ("Rene Descartes"). Despite a lack of formal training in mathematics, he is known now as the father of analytical geometry due to his discoveries in the fields of geometry and algebra and his written work *Geometry* (Moorman 299).

It seemed unlikely that Descartes, born March 31, 1596 in the town of La Haye in the south of France, would come upon excellence in the field of mathematics. His grandfather and great-grandfather had been doctors. His father was a lawyer and councilor in Parliament (Sorrell 6). From his degrees in law and the conception he held while attending the Jesuit school that mathematics is "good only for clever tricks and mean trades," one can speculate that he intended to follow in his father's footsteps (Jones 47). These ideas seem to have changed while Descartes was in the army of Prince Maurice of Orange. While touring with the army in Breda, Descartes met Isaac Beeckman, who agreed to translate a Dutch placard if Descartes would then answer it. Unknown to Descartes, the placard was a challenge to solve a geometrical

problem. Within a few hours, Descartes was able to work out the problem. Following this encounter, Descartes continued his practice of mathematics during his participation in the Thirty Years War. On the night of November 10, 1619, while campaigning on the Danube, Descartes experienced dreams that he claimed bore his first ideas for his new philosophy and analytical geometry. Shortly following this epiphany, in 1621, Descartes resigned his commission in the army to travel and study pure mathematics (Ball).

His study of pure mathematics continued for five years. In 1628, Cardinal de Berulle commissioned Descartes to dedicate his life to the "examination of truth" (Ball). Descartes spent the following twenty years studying philosophy and mathematics. During this time he developed a written work entitled *Regulae*, in which he believed to have discovered a method for solving any problem concerning number and figure. To compliment this, he developed a procedure that could transcribe any scientific problem to either number or figure, thus making it solvable by his method (Sorrell 19). In this case, Descartes' ambitions got the best of him, for his method and procedure were not adequate to fulfill their claim. His most notable work to come from this time period would be *Geometry* in 1637 ("Rene Descartes").

In *Geometry*, Descartes explores the theory of analytical geometry in a collection of three books. The first discusses a problem proposed by the Greek Pappus and Descartes' work with it, which led to his invention of analytical geometry, the study of geometry through the application of principles of algebra. The second book focuses on Descartes' studies of geometrical and mechanical curves. The third book ends the collection by focusing on an analysis of algebra (Ball).

The problem proposed by Pappus, included in the first book of *Geometry*, focuses on finding the locus of a point so the product of the perpendiculars on *m* given straight lines will maintain a constant ratio to the product of the perpendiculars on *n* other given straight lines. If *m=n*=2, Pappus explained, then the locus is a conic. Descartes was able to show the curve was represented by an equation of the second degree, which is a conic, but was unable to prove this by pure geometry (Ball).

In the second book of *Geometry*, Descartes defines geometrical curves to be "those which can be generated by the intersection of two lines each moving parallel to one coordinate axis with 'commensurable' velocities" (qtd. in Ball). In other words, *dy/dx* is an algebraic function. Descartes defined a curve to be mechanical "when the ratio of the velocities of these lines is 'incommensurable'," which means that *dy/dx* is a transcendental function (qtd. in Ball).

In the third portion of *Geometry*, which concerns the analysis of algebra, Descartes introduced rational devices for clarity in mathematics: He invented the use of *x*, *y*, and *z* to represent unknown values and *a*, *b*, and *c* to represent known values in equations (Sorrell 19). This writing also depicts Descartes as the first to understand advantages of moving all terms of an equation to one side. He also realized the meaning of negative quantities and made use of the rule for finding the limit to the number of positive and negative roots of an algebraic equation. He introduced the method of indeterminate coefficients for the solutions of equations in this portion of text as well (Ball).

Adding to clarity in mathematics, Descartes pioneered standard notations for cubes and higher powers and their corresponding roots. Descartes showed how lines in geometry could represent all quantities with expressible relations and how lines and curves were expressible

through algebraic notation (Sorrell 19). In order to work with new geometric curves, Descartes invented a machine that could draw curves previously thought indescribable strictly by geometric techniques, which he labeled as mechanical curves. He then displayed how these mechanic curves could be assimilated into geometric ones and how these lines and figures could be represented by equations (Sorrell 43).

Specifically, Descartes' greatest discovery was that a point in a plane could be determined given distances of x and y from two fixed lines of a right angle in the plan and that f(x,y)=0 was indeterminate and could be satisfied by infinite values of x and y. Then these points, x and y, determined the coordinates of multiple points that form a curve, which the function f(x,y)=0 expresses as a geometrical property true of the curve at every point on it. While Descartes' interests remained on plane curves, it follows that every point in space could similarly be determined by use of three coordinates. Descartes also improved upon what earlier writers had hinted at by concluding that two or more curves can be referred to one and the same system of coordinates. He went further still, stating that the points where two curves intersect can be determined by finding the roots common to their two equations (Ball).

Being both a great mathematician and thinker, it is only natural that Descartes would parlay his powers of mathematics into other sciences. He made great strides in the field of optics, publishing a work entitled *Dioptrics*. By use of his geometric knowledge of curves he studied the formation of rainbows and refraction of light, optimal curves for telescope and microscope lenses, and the makeup of the eye (Sorrell 38-40). Descartes also developed a universal mathematics he called "mathesis," which he stated, "explains all the points that can be raised concerning order and measure irrespective of the subject matter" (Sorrell 14). He

created a theory of vortices, which was the first attempt to give a complete mathematical and mechanical explanation of the universe as if it were a machine (Moorman 305). Descartes also deduced a theory of gravity concerning the planets and their satellites' evolutions. It was based on the vortex theory: a whirlpool was centered on the sun, around which planets swept. This same theory was applied to the moon to explain its revolutions about the earth. There were flaws in his theory, due to both physics that didn't consider resistance or mass in calculations, and the fact that he rarely tested his ideas experimentally (Sorrell 91). Although his ideas of planetary motion were innovative, he hesitated to publish them for fear he would offend the Inquisition (Sorrell 35).

Though Rene Descartes made large contributions to the field of mathematics, the greater portion of his work falls into the field of philosophy. Unlike most philosophers, though, he took a mathematician's approach to thought, considering analysis to be more important a technique than synthesis (Moorman 299). His mathematical prowess effectively made his philosophy as powerful as it was and continues to be. He composed three important texts of philosophy: *Discourse on the Method of Rightly Conducting the Reason and Seeking Truth in the Sciences*, *Meditations on First Philosophy*, and *Principles of Philosophy* ("Rene Descartes").

Beyond all his accomplishments lies what Descartes describes as his greatest sorrow: the death of his 5-year-old illegitimate daughter Francine in 1640 (Sorrell 56). Only 10 years later, at age 54, Descartes himself would pass away. He was a notoriously weak individual, requiring that he remain in bed until he felt able to rise. This most likely contributed to his contraction of pneumonia while tutoring the Queen of Switzerland, who required her learning to begin at five in the morning ("Rene Descartes"). Having little family or other cares, Descartes

used the majority of his time to dream up genius ideas of mathematics and other sciences. Though he has been dead for over three centuries, the popularity of his work will not allow his body to receive the rest it needs. Disputes continue over where his final burial ground should be. In fact, his skull is on display at the Musee de l'Homme in Paris (Sebba 225).

Works Cited

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Theorems

For the ten theorems I explore, a few definitions should be understood for application. They are as follows:

Definition of Subset: Let A and B be sets. We say A is a subset of B and write A $\subseteq$ B iff ($\forall$x)(x $\in$ A $\Rightarrow$ x $\in$ B).

Definition of Intersection: Let A and B be sets. The intersection of A and B, denoted A$\cap$B, is the set A$\cap$B = {x| x $\in$ A $\wedge$ x $\in$ B}.

Definition of Union: Let A and B be sets. The union of A and B, denoted A$\cup$B, is the set A$\cup$B = {x| x $\in$ A $\vee$ x $\in$ B}.

Definition of Disjoint: Let A and B be sets. We say A and B are disjoint iff A$\cap$B = $\emptyset$

Definition of Power set on set A: Let A be a set. The power set of A, denoted P(A), whose elements are the subsets of A, is defined as P(A)={B|B$\subseteq$A}.

Definition of Inverse: Let R be a relation. Then the inverse of R, denoted R-1, is the set R-1 = {(y,x)| (x,y) $\in$ R}.

Definition of Symmetry: Let R be a relation. Then R is symmetric if ($\forall$x,y)(xRy $\Rightarrow$ yRx).

Definition of the Composite of R and S: Let R and S be relations from sets A to B and B to C respectively. The composite of R and S, denoted S°R, is the relation S°R = {(x,z)|($y)(xRy $\wedge$ ySz)}.

Inherent and explicit use of these definitions will be explored in a variety of theorems mainly involving set theory.

1) Theorem: For any sets A,B, A$\cap$B$\subseteq$ A.
   Proof:
   > Let A, B be arbitrary sets. Let x be arbitrary. Let x$\in$A$\cap$B. By the definition of intersection x$\in$A and x$\in$B. Therefore, x$\in$A. Since x was arbitrary, A$\cap$B$\subseteq$ A by the definition of subset. Therefore, for any sets A,B, A$\cap$B$\subseteq$ A, QED.

Explicit use of the definitions of intersection and subset enable the theorem A$\cap$B$\subseteq$ A to be proven. From this first theorem, one can see how these definitions will be used with one other. The second theorem in this series displays how understanding the definitions of subset and intersection can be used to prove that subsets of two disjoint sets are also disjoint.

2) Theorem: For any sets A, B, C, D, if C⊆A and D⊆B and A and B are disjoint, then C and D are disjoint.

Proof (by contraposition):

> Let A, B, C, D be arbitrary sets. Suppose C⊆A and D⊆B and C∩D≠Ø. Let x∈C∩D be arbitrary. Then x∈C and x∈D. Since C⊆A and D⊆B, x∈A and x∈B by the definition of subset. Therefore x∈A∩B. Since x was arbitrary, A∩B≠Ø. By contraposition, it follows that, for any sets A, B, C, D, if C⊆A and D⊆B and A and B are disjoint, then C and D are disjoint, QED.

So far the definitions of subset, intersection, and disjoint have each been applied in the first two proofs. Now consider their use to prove an if and only if condition. We can apply previous experiences to theorem three; the work done proving A∩B⊆ A in theorem one will be useful.

3) Theorem: For any sets A, B, A⊆B ⟺ A∩B=A

Proof:

> <First we must prove A⊆B ⟹ A∩B=A.> Let A, B be arbitrary sets. Suppose A⊆B. Let x∈A be arbitrary. From hypothesis, A⊆B. Since x∈A, then x∈B by the definition of subset. Hence, x∈A implies x∈A and x∈B, so x∈A∩B. Because x∈A and x∈A∩B, then A∩B⊆A. Thus, A∩B=A.

> <Next we must prove A∩B=A ⟹ A⊆B.> Suppose A∩B=A. Let x∈A∩B. Then x∈A and x∈B by the definition of intersection. Thus x∈B and x∈A, which is equivalent to x∈B∩A. From a previous theorem A∩B⊆A, so obviously B∩A⊆B. Therefore B∩A= A∩B. It follows that A∩B⊆B. By hypothesis, A∩B=A, so A⊆B, and if A⊆B then A∩B=A and if A∩B=A then A⊆B. By that reason, for any sets A,B A⊆B ⟺ A∩B=A, QED.

Another example of a bi-conditional proof follows, but rather than assert that A⊆B ⟺ A∩B=A, it proves how A⊆B ⟺ A∪B =B. This is a slight difference that will explore how the union of two sets will change our proof. Though it is not explored within these 10 proofs, it is logical to reason by use of proofs number 3 and 4 that A∩B=A⟺ A∪B =B.

4) Theorem: For any sets A, B, A⊆B ⟺ A∪B =B.

Proof:

> <First show that A∪B = B ⟹ A⊆B.> Let A,B be arbitrary sets. Suppose A∪B = B. Let x be arbitrary. Suppose x∈A. Then certainly x∈A or x∈B. It follows that x∈A∪B. Since A∪B = B, x∈B. Since x was arbitrary, A⊆B. Therefore A∪B = B ⟹ A⊆B.
> <Next show that A⊆B ⟹ A∪B =B.> Suppose A⊆B. Let x be arbitrary. Suppose x∈A∪B. By the definition of union, x∈A or x∈B. If x∈B, we're finished. If x∈A, since A⊆B, x∈B. In either case, x∈B. Therefore, since x was arbitrary, A∪B ⊆B. From a previous theorem, B⊆A∪B. Thus, A⊆B ⟹ A∪B =B. Since A, B were arbitrary, A⊆B ⟺ A∪B =B for all sets A,B, QED.

Intersection was used in the proof of A⊆B ⇔ A∩B=A. When the assumption is made that a set is the subset of the intersection of two sets, it can be seen how that set must be a subset of each of the two intersecting sets both by transitivity and the definition of intersection. The proof of this intuitive knowledge follows.

5) Theorem: C⊆A∩B ⇔ C⊆A ∧ C⊆B
   Proof:
   <First prove C⊆A∩B ⇒ C⊆A ∧ C⊆B.> Let A, B, C be arbitrary sets. Suppose C⊆A∩B. Let x∈C be arbitrary. Then, by definition of subset, x∈A∩B. Consequently, C⊆A ∧ C⊆B.

   <Next prove C⊆A ∧ C⊆B ⇒ C⊆A∩B.> Suppose C⊆A ∧ C⊆B. Let x∈C be arbitrary. Then, by the definition of subset, x∈A and x∈B. Since x was arbitrary, x∈A∩B. Hence, C⊆A ∧ C⊆B. Thus, for any sets A, B, C, C⊆A∩B ⇔ C⊆A ∧ C⊆B, QED.

Since our definitions can be manipulated by applying the use of an arbitrary *x* element, one may explore how the application of an arbitrary *x* element will look in a proof by cases.

6) Theorem: For any sets A,B,C, if A⊆C and B⊆C, then A∪B⊆C
   Proof:
   Let A,B,C be sets. Suppose A⊆C and B⊆C. Let x∈A∪B be arbitrary. Then x∈A or x∈B by the definition of Union.

   Case 1: If x∈A, then by the definition of subset, x∈C.
   Case 2: If x∈B, then by the definition of subset, x∈C.

   Therefore, since x∈C in both cases, and since x was arbitrary, A∪B⊆C by the definition of subset. Thus, for any sets A,B,C, if A⊆C and B⊆C, then A∪B⊆C, QED.

Up to this point, extensive work has been done with subsets. In the proof of theorem 7, we will continue working with subsets in relation to the power set.

7) Theorem: For sets A, and B, A⊆B⇔P(A) ⊆ P(B).
   Proof:
   Let A and B be sets. Let P(A) and P(B) be the power sets of A and B respectively. Suppose A⊆B and suppose x∈P(A). <Must show x∈P(B).> x∈P(A) implies that x⊆A. Since A⊆B, then x⊆B. x⊆B implies that x∈P(B). Accordingly, x∈P(A) implies x∈P(B). Thus P(A) ⊆P(B), QED.

Union is applied to symmetric and inverse relations. When the definition of union and symmetry are known, it is intuitively obvious that the union of the relation and its inverse will be symmetric.

8) Theorem: Let R be a relation. Then $R \cup R^{-1}$ is symmetric.
   Proof:

   > Let R be a relation on set A. Suppose $(x,y) \in R \cup R^{-1}$. Then $(x,y) \in R$ or $(x,y) \in R^{-1}$. If $(x,y) \in R$, then $(y,x) \in R^{-1}$. Similarly, if $(x,y) \in R^{-1}$, then $(y,x) \in R$. In either case, $(y,x) \in R \cup R^{-1}$. Thus $R \cup R^{-1}$ is symmetric, QED.

Since inverse relations and symmetry have been considered, it will be easier to apply these concepts in a more involved proof. The following bi-conditional theorem calls for further use of inverse relations and symmetry, as well as the use of subset.

9) Theorem: R is symmetric $\Leftrightarrow R^{-1} \subseteq R$.
   Proof:

   > <First we show R is symmetric $\Rightarrow R^{-1} \subseteq R$.> Let A be any set and R be a relation on A. Suppose R is symmetric. Let $(x,y) \in R^{-1}$ be arbitrary. By the definition of inverse, $(y,x) \in R$. Since R is symmetric, by hypothesis, $(x,y) \in R$. Thus, since (x,y) was arbitrary, $R^{-1} \subseteq R$.

   > <Next show $R^{-1} \subseteq R \Rightarrow R$ is symmetric.> Suppose $R^{-1} \subseteq R$. Let $(x,y) \in R$ be arbitrary. By the definition of inverse, $(y,x) \in R^{-1}$. By the definition of symmetry, $(y,x) \in R^{-1} \subseteq R$. Since (x,y) was arbitrary, R is symmetric. Since A and R were arbitrary, $\forall R \in A$, R is symmetric $\Leftrightarrow R^{-1} \subseteq R$, QED.

Now a better understanding of symmetry and inverse relations has been acquired. Consider the use of composition with inverse. The following theorem manipulates said concepts in a repetitious fashion to determine the proof.

10) Theorem: $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$.
   Proof:

   > <First show $(S \circ R)^{-1} \subseteq R^{-1} \circ S^{-1}$.> Let S and R be arbitrary relations. Let (x,z) be an arbitrary element of $(S \circ R)^{-1}$. By definition of inverse, $(z,x) \in S \circ R$. By definition of composition, $\$ y$ such that $(z,y) \in R$ and $(y,x) \in S$. By definition of inverse $(y,z) \in R^{-1}$ and $(x,y) \in S^{-1}$. By definition of composition $(x,z) \in R^{-1} \circ S^{-1}$. Since (x,z) was arbitrary, $(S \circ R)^{-1} \subseteq R^{-1} \circ S^{-1}$.

   > <Next show that $R^{-1} \circ S^{-1} \subseteq (S \circ R)^{-1}$.> Let (u,w) be an arbitrary element of $R^{-1} \circ S^{-1}$. By definition of composition, $\$ v$ such that $(u,v) \in S^{-1}$ and $(v,w) \in R^{-1}$. By definition of inverse, $(w,v) \in R$ and $(v,w) \in S$. By definition of composition, $(w,u) \in (S \circ R)$. By definition of inverse, $(u,w) (S \circ R)^{-1}$. Since (u,w) was arbitrary, $R^{-1} \circ S^{-1} \subseteq (S \circ R)^{-1}$. Since S and R were arbitrary, $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$. Hence, $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$, QED.

The theorems of this paper explore only a few of the many possible combinations of the listed definitions in reference to sets and relations. Hopefully this short exposition has provided a

practical understanding of the investigated concepts through the proving of the supplied theorems.

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